## LESSON DETAILS

### Algebraic Detectives - The Search for Algebraic Properties Through Patterning

### Lesson Summary

This lesson introduces students to some of the basic algebraic skills that will be used in this and future courses. Students will use CAS (Computer Algebra Systems) to test a variety of operations and look for patterns and determine algebraic properties. Then students will use code to test the properties, as well as to modify existing code to observe different properties. Throughout the lesson, students will identify and utilize resources and supports that aid in perseverance in mathematical learning and develop critical mathematical thinking.

### Grade: 9

### Big Ideas

Exploring algebraic expressions and operations through patterning

### Learning Expectations

**AA1.** develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands

- identifying resources and supports that aid perseverance in mathematical learning
- developing a healthy mathematical identity through building self-awareness
- develop critical and creative mathematical thinking skills

**A1.** apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- reasoning and proving
- representing
- selecting tools and strategies

**A2.** make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

**B2.** represent numbers in various ways, evaluate powers, and simplify expressions by using the relationships between powers and their exponents

**B2.2** analyse, through the use of patterning, the relationships between the exponents of powers and the operations with powers, and use these relationships to simplify numeric and algebraic expressions

**C1.** demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

**C1.4** simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts

**C2.** apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands

**C2.1** use coding to demonstrate an understanding of algebraic concepts including variables, parameters, equations, and inequalities

## Cross Curricular Connections

## Learning Goals and Success Criteria:

LG1: We are learning to use patterns to explore properties of algebraic expressions.

- SC1: I can use CAS to identify patterns in algebra.
- SC2: I can explain how to collect like terms.
- SC3: I can explain how to multiply and divide powers with the same base.
- SC4: I can explain how to simplify powers of powers.
- SC5: I can explain how to use the distributive property to simplify algebraic expressions.

LG2: We are learning to utilize the correct algebraic properties to simplify expressions.

- SC1: I can make connections between the type of algebraic expression given and the correct strategy necessary to simplify the expression.
- SC2: I can explain the similarities and differences of the various strategies to use when simplifying expressions.
- SC3: I can select the correct tools and strategies to simplify expressions.

LG3: We are learning to interpret and modify text based code to represent algebraic properties.

- SC1: I can use code to simplify expressions that involve multiplying powers with the same base. (As in the success criteria checklist, it may be beneficial not to share this wording with the class, instead stating "I can use provided code to determine what the code does." This will allow students to identify the purpose of the code.)

SC2: I can modify code to perform other operations with powers.

## CONSIDERATIONS THROUGHOUT THE LESSON

### Differentiated Instruction and Universal Design for Learning

Use Visibly Random Groupings to create small groups.
Use flexible small group instruction for students who need support.
Use visuals of patterns, technology, and worksheets to display a variety of methods of observing algebraic expressions.
Provide parallel questions for all stations.
Provide extension questions for consolidation and practice.
Previewing vocabulary for English Language Learners and students with special education needs would be beneficial. (Examples of terms that may be included: algebra, powers, base, variable, distribute, “like” terms, simplify ).
Use assistive technology or alternate formats (ex. verbal) for the exit card.

### Assessment

Throughout the lesson, the teacher will be listening for students correctly and effectively using mathematical language to describe their algebraic thinking.

Checkpoints - Testing out their thinking after each station (see Action section)
Are students seeing the similarities and differences of simplifying each type of algebraic expression? Can students explain how to collect like terms and use the distributive property? Can students explain how to multiply and divide powers with the same base? Can students explain how to simplify powers of powers? Can students explain how to apply the distributive property to algebraic expressions?

Students will self evaluate using the Success Criteria.

A complete set of clues at the end of all stations indicates completion and some mastery.

Combination questions (see Consolidation section)

Exit Card (see Consolidation section)

## RESOURCES AND LEARNING ENVIRONMENT

### Educator Resources Needed

Access to Computer Algebra Systems (CAS)
Access to coding device (Python examples provided)
Prepared pictures, station, clue, puzzle and consolidation question pages (provided)

### Student Materials Needed

Technology (Computer Algebra Systems (CAS) and Python compiler)

### Learning Environment Considerations

During this lesson, students will be asked to focus on identifying resources and supports that aid perseverance in mathematical learning.
From the Curriculum Context, identifying resources and supports that aid perseverance in mathematical learning will include:

- Embracing mistakes as a necessary and helpful part of learning
- Noticing strengths and positive aspects of experiences, appreciating the value of practice
- Creating a list of supports and resources, including people, that can aid them in persevering
- Applying strategies such as:
  - supporting peers by encouraging them to keep trying if they make a mistake
  - using personal affirmations like “I can do this.”

The Minds-On section starts with students working in small groups (2-4 students). Each group will need visuals of different types of items to be grouped by patterning. This could be done on VNPS (Vertical Non-Permanent Surfaces) or at desks. Then consolidate this concept in a large group before moving to the Action section.

In the Action section, students will work in small groups to complete the algebra station activities. They may work individually, in pairs or small groups (depending on availability of resources) to complete the coding activities.

For the Consolidation section, it would be ideal to use VNPS (Vertical Non-Permanent Surfaces).

Previewing vocabulary for students who are English Language Learners and students with special education needs would be beneficial. (Examples of terms that may be included:

algebra, powers, base, variable, distribute, “like” terms, simplify ). The use of a word wall with visuals would be one way to ensure this is in place and would provide ongoing support that benefits all learners.

Consider “de-fronting” the classroom, allowing students to sit in groups rather than in rows facing the front of the room.  This helps send the message to students that the space is safe and it is okay to not be perfect.

## LESSON CONTENT

**Minds-On** (15-20 minutes)

There are two different sets of pictures that have images that can be grouped provided in the appendix.  You can create others if you wish for more variety.  For each set of pictures, there are a number of ways each of the images can be grouped. Have students “meet” in small groups to begin the grouping and discussion. You may choose to have each group get all of the pictures, or perhaps a few groups get the same pictures and other groups get different pictures.
Then bring the entire class together to share their observations and have a discussion. It is recommended that you discuss how and why the same items can meet more than one type of grouping criteria. For example with the balls: they can be grouped by type, or grouped by those that are kicked (soccer and football), or thrown (baseball and golf), or grouped by the shapes (round or not round), or the colour, etc. In the second set, the 3 pictures of bananas might be grouped together, and the 3 flowers might be grouped together, but all 6 might be grouped together as "things that grow", or students might group the image that shows 2 flowers with the image that shows 2 scoops of ice cream and the image that shows two connected cubes. Students should be encouraged to think of many different ways to group these images. This is an opportunity to have your students reflect on the attributes they have, and in how many ways they may be “grouped”. This will allow students to reflect on how complex they are and to develop a healthy mathematical identity through building self-awareness.

Set up the premise of the lesson.  Students will be detectives trying to discover the secrets of algebra.  They will be given patterns to try to determine a rule and will have an opportunity to test their hypothesis against trial questions. Discuss with the class that in algebra, as in life, expressions also have certain attributes that allow them to be grouped. Although algebraic expressions may start out looking different, once simplified they may look the same. These expressions can be grouped as being equivalent.
You could make this into a game by having trial questions uncover clues that lead to a final discovery (an example is included below the lesson, in the appendix).

**Action** (75-90 minutes)

Six stations of patterns for algebraic exploration have been created and are in the appendix below. Each station contains 10 - 12 questions of increasing complexity. Depending on the size of the class, you may need to have multiple copies of each station. The topics for the first five stations include: Collecting Like Terms (grade 8 but worth reviewing), Distributive Property with Numbers, Multiplying Powers with the Same Base, Dividing Powers with the Same Base, and Power of a Power.
Divide students into random groups of three. At each station students will work together or separately (depending on availability of technology) to enter the provided questions into CAS and then look for the answer that CAS provides. Once students complete all provided examples, they are to collaboratively determine the pattern/rule/relationship observed.

When a group is confident in their "evidence" (understanding) they will ask the teacher for clue questions. This is a Check Point - Clue Questions are in the appendix as is a checklist for the success criteria. These should be done WITHOUT the use of CAS. The teacher can decide whether to make this practice individual or in groups. If the students are not correct, you may choose to have them review their previous work and try a second Clue Question set. If you feel it is necessary that they repeat the patterning activity, a second version of the station has been created for you in the appendix.

Please note, in the "Dividing Powers with the Same Base" station, some of the exponents are negative. This is to explicitly show that one must subtract a negative. Often students believe that the negative sign and the subtraction are interchangeable rather than consecutive.

If students are correct, give them whatever "clues" you have determined they have found. In the example provided in the appendix, you may tell them that "All #1 are "y"s, #2 are "o"s and 3# are "u"s. Students add these letters to their puzzle. Student groups then move on to the next station. The order in which these stations are completed is not important.

A sixth station of Distributive Property with Variable Terms needs to be completed only after the other five are done, as it combines distributive property and the exponent rules. As mentioned, if any group is struggling to determine or apply one of the concepts, you have access to a second set of all stations for continued patterning practice.

When individual groups have completed all stations and have tested all of their knowledge against your clue questions, students can progress to the coding activity, still in their small groups.

The coding portion of this lesson will include a provided set of text code for multiplying powers having the same base. Students will be asked to determine which of the types of

algebra we just worked with is represented by the provided code.  They should then run the code to ensure their hypothesis is correct and to observe the formatting of the results. Have the students run the code with variable bases. This can be done either using Python on TI or using a free Python compiler on a computer.  The teacher will check in with students to ensure there is understanding of this first step in the coding activity.  The teacher will then instruct the students to modify the code to now divide powers having the same base.  Once students have made the modifications and run the new code, their final coding task will be to modify the code to determine the Power of a Power.
A screen print of an example of a Python code has been provided in the appendix, as has a screen print of the modified sections of the next two coding tasks.

**Extension Opportunity:**
If any groups are significantly faster than the rest of the class, the teacher could challenge these students to attempt to create code for one of the other concepts explored with CAS.

**Consolidation** (10 minutes & 20 minutes)

Since this is a lesson that cannot be completed during most regular length classes, consolidation will occur twice.  The first consolidation will occur after all groups have completed the Stations activity (or at the end of the period or beginning of the next period, as appropriate).  The second consolidation will occur at the end of the entire lesson.

Consolidation 1:
As a class, create a consolidated list of rules that have been identified to address each of the situations explored.  Ensure that the similarities and differences of the strategies necessary to solve these algebraic expressions are discussed.  These techniques will be necessary as we move through the course and will be utilized in a variety of situations. Teachers may then choose to have students create their own notes, create a class anchor chart, etc.  After this consolidation, groups can either begin or continue the coding activity.

Consolidation 2:
Following the coding activity, students will have an opportunity to test their understanding of all concepts.  The teacher will provide a combination question set for consolidation as other groups complete the coding.  This combination set would work very well on Vertical Non Permanent Surfaces, but could also be done at student desks. A combination set of questions has been provided in the appendix.  Also provided is a parallel set of questions (similar degree of difficulty) as well as some extension questions that could be used depending on the competency and confidence of the groups.  This consolidation corresponds to the last two success criteria on the checklist.

Exit Card Option:

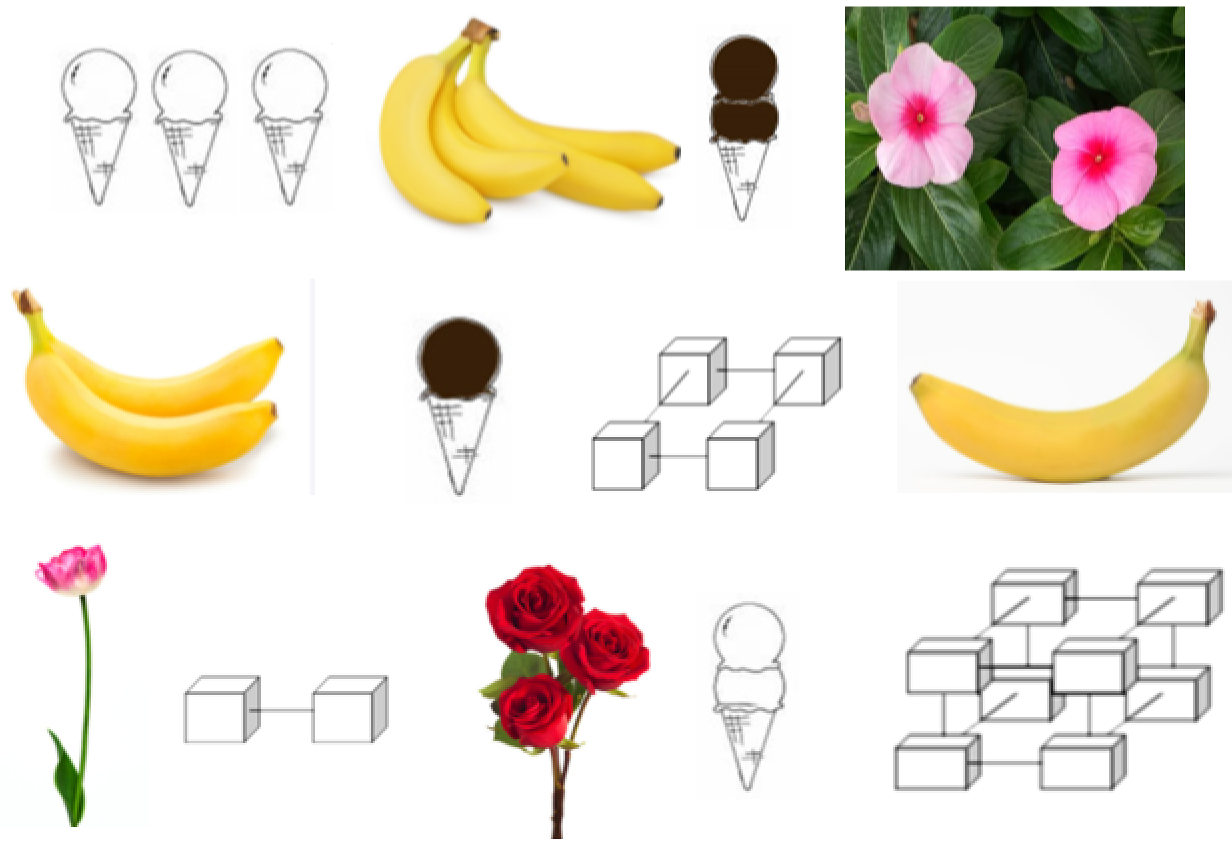
Have the students create and answer one question of each type that we have explored in this lesson. An example is provided in the appendix. Keep in mind that this can be created in a variety of forms and environments.

## APPENDIX:

### Minds-On Sample Pictures:

### Ball-Types Pictures:

**Mixed-Type Pictures:**



**Patterning Stations:**

**Collecting Like Terms Version 1:**

| **Collecting Like Terms** | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $6x + 2x$ | |
| $6x + 4x$ | |
| $6x - 4x$ | |
| $6x - 5x$ | |
| $6x - x$ | |
| $6x + x$ | |
| $6x + x + 4$ | |
| $6x + x + 4 + 3$ | |
| $6x - x + 4 - 3$ | |
| $6x + 4 - x - 3$ | |
| $8x + 4 + 2x - 3$ | |
| $8x - 6 + 2x - 4$ | |
| **When collecting like terms, I…** | |

**Collecting Like Terms Version 2:**

| Collecting Like Terms | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $7x + 3x$ | |
| $7x + 5x$ | |
| $7x - 5x$ | |
| $7x - 6x$ | |
| $7x - x$ | |
| $7x + x$ | |
| $7x + x + 5$ | |
| $7x + x + 5 + 4$ | |
| $7x - x + 5 - 4$ | |
| $7x + 5 - x - 4$ | |
| $5x + 5 + 3x - 4$ | |
| $5x - 7 + 3x - 2$ | |
| **When collecting like terms, I…** | |

**Distributive Property with Constant Coefficients Version 1:**

| Distributive Property | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $2(x+3)$ | |
| $2(4x+1)$ | |
| $2(4x-1)$ | |
| $2(4x-3)$ | |
| $-2(4x-3)$ | |
| $-2(3x+3)$ | |
| $-2(-3x+3)$ | |
| $-2(-3x-3)$ | |
| $-2(-3x-3)+4(x-5)$ | |
| $-2(-3x-3)+4(2x-5)$ | |
| $-2(-3x-3)-4(2x-5)$ | |
| $-2(-3x-3)-4(2x+5)$ | |
| **When using the distributive property, I…** | |

**Distributive Property with Constant Coefficients Version 2:**

| Distributive Property | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $3(x+1)$ | |
| $3(2x+1)$ | |
| $3(2x-1)$ | |
| $3(2x-5)$ | |
| $-3(2x-5)$ | |
| $-3(4x+5)$ | |
| $-3(-4x+5)$ | |
| $-3(-4x-5)$ | |
| $-3(-4x-5)+2(x+6)$ | |
| $-3(-4x-5)+2(7x+6)$ | |
| $-3(-4x-5)-2(7x+6)$ | |
| $-3(-4x-5)-2(7x-6)$ | |
| **When using the distributive property, I…** | |

**Multiplying Powers with the Same Base Version 1:**

| Multiplying Powers with the Same Base | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $x \cdot x$ | |
| $x \cdot x \cdot x \cdot x$ | |
| $x^2 \cdot x$ | |
| $x^2 \cdot x^3$ | |
| $x^4 \cdot x^3$ | |
| $x^2 y \cdot x^2 y$ | |
| $x^2 y \cdot x^3 y^2$ | |
| $x^4 \cdot x^3 \cdot x$ | |
| $x^3 y \cdot x^2 \cdot x y^2$ | |
| $x^4 y^2 \cdot x^7 y^3$ | |
| $x^4 \cdot y^3 \cdot z^2$ | |
| $x^2 y^3 \cdot x^2 z^3 \cdot y^2 z^3$ | |
| **When multiplying powers with the same base, I…** | |

**Multiplying Powers with the Same Base Version 2:**

| Multiplying Powers with the Same Base | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $x \cdot x \cdot x$ | |
| $x \cdot x \cdot x \cdot x \cdot x$ | |
| $x^2 \cdot x \cdot x$ | |
| $x^2 \cdot x^2$ | |
| $x^4 \cdot x^2$ | |
| $x^4 y \cdot x^2 y$ | |
| $x^4 y \cdot x^3 y^2$ | |
| $x^4 \cdot x^2 \cdot x$ | |
| $x^4 y \cdot x^2 \cdot xy$ | |
| $x^4 y^2 \cdot x^{12} y^2$ | |
| $x^2 \cdot y^3 \cdot z^4$ | |
| $x^3 y^3 \cdot x^3 z^3 \cdot y^3 z^3$ | |
| **When multiplying powers with the same base, I…** | |

**Dividing Powers with the Same Base Version 1:**

| Dividing Powers with the Same Base | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $x^2 \div x$ | |
| $x^3 \div x$ | |
| $\frac{x^4}{x}$ | |
| $x^4 \div x^2$ | |
| $x^7 \div x^2$ | |
| $x^7 y^4 \div x^2 y$ | |
| $x^7 y^5 \div x^2 y^3$ | |
| $x^7 \div y^3$ | |
| $x^7 y^5 z^4 \div x^2 y^3 z$ | |
| $x^7 y^5 z^4 \div x^2 z$ | |
| $x^2 \div x^{-2}$ | |
| $x^2 \div x^{-4}$ | |
| **When dividing powers with the same base, I…** | |

**Dividing Powers with the Same Base Version 2:**

| Dividing Powers with the Same Base | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $x^3 \div x$ | |
| $x^4 \div x$ | |
| $\frac{x^5}{x}$ | |
| $x^5 \div x^2$ | |
| $x^6 \div x^2$ | |
| $x^6 y^4 \div x^2 y^2$ | |
| $x^6 y^4 \div x^2 y^3$ | |
| $x^6 \div y^3$ | |
| $x^6 y^7 z^5 \div x^2 y^3 z$ | |
| $x^6 y^7 z^5 \div y^3 z$ | |
| $x^5 \div x^{-2}$ | |
| $x^5 \div x^{-5}$ | |
| **When dividing powers with the same base, I…** | |

**Power of a Power Version 1:**

| Power of a Power | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $\left(x^2\right)^2$ | |
| $\left(x^2\right)^4$ | |
| $\left(x^3\right)^4$ | |
| $\left(x^2 y\right)^2$ | |
| $\left(x^2 y^3\right)^2$ | |
| $\left(x^4 y^3\right)^4$ | |
| $\left(x^4 y^3 z^3\right)^4$ | |
| $\left(\frac{x^2}{y}\right)^2$ | |
| $\left(\frac{x^4}{y^2}\right)^2$ | |
| $\left(\frac{x^4 z^3}{y^2}\right)^2$ | |
| **When simplifying a power of a power, I…** | |

**Power of a Power Version 2:**

| Power of a Power | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $\left(x^2\right)^3$ | |
| $\left(x^2\right)^5$ | |
| $\left(x^4\right)^5$ | |
| $\left(x^2 y\right)^3$ | |
| $\left(x^2 y^3\right)^3$ | |
| $\left(x^4 y^3\right)^5$ | |
| $\left(x^4 y^3 z^2\right)^5$ | |
| $\left(\frac{x^2}{y}\right)^3$ | |
| $\left(\frac{x^4}{y^2}\right)^3$ | |
| $\left(\frac{x^4 z^5}{y^2}\right)^3$ | |
| **When simplifying a power of a power, I…** | |

**Distributive Property with Variable Coefficients Version 1:**

| Distributive Property | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $2x(x+3)$ | |
| $2x(4x+1)$ | |
| $2x\left(4x^2-1\right)$ | |
| $2x\left(4x^2-3\right)$ | |
| $-2x^2(4x-3)$ | |
| $-2x^2(3x+3)$ | |
| $-2x(-3x+3)$ | |
| $-2\left(-3x-3x^2\right)$ | |
| $-2x(-3x-3)+4x(x-5)$ | |
| $-2x(-3x-3)+4x(2x-5)$ | |
| $-2x^2(-3x-3)-4x(2x-5)$ | |
| $-2x^2(-3x-3)-4x(2x+5)$ | |
| **When using the distributive property, I…** | |

**Distributive Property with Variable Coefficients Version 2:**

| **Distributive Property** | |
|---|---|
| **Input (Question)** | **Output (Answer given by CAS)** |
| $3x(x+1)$ | |
| $3x(2x+1)$ | |
| $3x\left(2x^2-1\right)$ | |
| $3x\left(2x^2-5\right)$ | |
| $-3x^2(2x-5)$ | |
| $-3x^2(4x+5)$ | |
| $-3x(-4x+5)$ | |
| $-3\left(-4x+5x^2\right)$ | |
| $-3x(-4x-5)+2x(x-6)$ | |
| $-3x(-4x-5)+2x(7x-6)$ | |
| $-3x^2(-4x-5)-2x(7x-6)$ | |
| $-3x^2(-4x-5)-2x(7x+6)$ | |
| **When using the distributive property, I…** | |

**Station Clue Questions: (without CAS)**

| Collecting Like Terms Clue Questions | | Collecting Like Terms Clue Questions | |
|---|---|---|---|
| **1** | $5x + 7x$ | **1** | $4x + 10x$ |
| **2** | $9x - 6 - 2x$ | **2** | $11x - 7 - 3x$ |
| **3** | $5x + 4 - 2x + 6$ | **3** | $7x + 3 - 4x + 2$ |

| Distributive Property Clue Questions | | Distributive Property Clue Questions | |
|---|---|---|---|
| **1** | $4(2x + 5)$ | **1** | $6(3x + 4)$ |
| **2** | $-5(x + 3)$ | **2** | $-4(2x + 5)$ |
| **3** | $-4(x - 2) - 2(2x + 7)$ | **3** | $-3(2x - 7) + 2(2x - 7)$ |

| Multiplying Powers with the Same Base Clue Questions | | Multiplying Powers with the Same Base Clue Questions | |
|---|---|---|---|
| 1 | $x^4 \cdot x^3$ | 1 | $x^2 \cdot x^3$ |
| 2 | $x^3 y \cdot x^2 y^2$ | 2 | $x^3 y \cdot x^4 y^3$ |
| 3 | $2^4 \cdot 2^3$ | 3 | $3^2 \cdot 3^4$ |

| Dividing Powers with the Same Base Clue Questions | | Dividing Powers with the Same Base Clue Questions | |
|---|---|---|---|
| 1 | $x^7 \div x^2$ | 1 | $x^6 \div x^4$ |
| 2 | $x^5 y^4 \div x^4 y$ | 2 | $x^3 y^4 \div x y^3$ |
| 3 | $2^5 \div 2^3$ | 3 | $3^6 \div 3^3$ |

| Power of a Power Clue Questions | | Power of a Power Clue Questions | |
|---|---|---|---|
| 1 | $\left(x^3\right)^2$ | 1 | $\left(x^4\right)^2$ |
| 2 | $\left(xy^2\right)^4$ | 2 | $\left(x^3y\right)^3$ |
| 3 | $\left(2^2\right)^4$ | 3 | $\left(3^2\right)^3$ |

| Distributive Property with Variable Terms Clue Questions | | Distributive Property with Variable Terms Clue Questions | |
|---|---|---|---|
| 1 | $2x(3x+4)$ | 1 | $3x(4x+4)$ |
| 2 | $-4x(2x-1)$ | 2 | $-2x(3x-4)$ |
| 3 | $-3x\left(-x^2+1\right)$ | 3 | $-x\left(-2x^2+1\right)$ |

**Sample Clue Puzzle:**

**Blank:**

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 6 | 8 | 6 | 9 | 2 | 10 | 11 | 12 | 13 |
| | | | | | | | | | | | | | | | | | |
| 4 | 9 | 13 | 6 | 14 | 5 | 4 | 11 | 18 | | 15 | 4 | 16 | 17 | 6 | 5 | 1 | |
| | | | | | | | | | | | | | | | | | |

**Solution:**

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 6 | 8 | 6 | 9 | 2 | 10 | 11 | 12 | 13 |
| **Y** | **O** | **U** | | **A** | **R** | **E** | | **D** | **E** | **V** | **E** | **L** | **O** | **P** | **I** | **N** | **G** |
| 4 | 9 | 13 | 6 | 14 | 5 | 4 | 11 | 18 | | 15 | 4 | 16 | 17 | 6 | 5 | 1 | |
| **A** | **L** | **G** | **E** | **B** | **R** | **A** | **I** | **C** | | **M** | **A** | **S** | **T** | **E** | **R** | **Y** | |

**Success Criteria Checklist:**

As you complete each section of the lesson, use this checklist to track your understanding.

| **Criteria** | I have got this! I had everything correct on the first attempt. | I get it now. I made some mistakes but figured it out after more practice. | I mostly get it but still need more practice. |
|---|---|---|---|
| **Algebra Patterns:** | | | |
| I can use CAS to identify patterns in algebra. | | | |
| I can explain how to collect like terms. | | | |
| I can explain how to multiply and divide powers with the same base. | | | |
| I can explain how to simplify powers of powers. | | | |
| I can explain how to apply the distributive property to algebraic expressions. | | | |
| **Coding:** | | | |
| I can use provided code to determine what the code does. | | | |
| I can modify code to perform other operations with powers. | | | |
| **Simplifying Expressions:** | | | |
| I can make connections between the type of algebraic expression given and the correct strategy necessary to simplify the expression. | | | |
| I can explain the similarities and differences of the various strategies to use when simplifying expressions. | | | |

**Consolidation Combination Questions: (without CAS)**

**Consolidation Combination Questions Version 1:**

| | |
|---|---|
| 1 | $x^3 y \cdot x^2 y^2$ |
| 2 | $3x + 7x^2 - 4x^2 + 6x$ |
| 3 | $3(7x - 4)$ |
| 4 | $x^3 \cdot x^7$ |
| 5 | $(y^3)^4$ |
| 6 | $-6(-2x + 4)$ |
| 7 | $x^8 y^5 \div x^4 y^3$ |
| 8 | $3xy + 7x^2 - 4x^2 y - 3x + x^2 - 3y$ |
| 9 | $3x(2x + 3) + 4x(3x + 1)$ |
| 10 | $y^7 \div y^2$ |
| 11 | $x^2 y \cdot x^5 y^3$ |
| 12 | $-6x(2x + 3)$ |
| 13 | $(x^3 y)^2$ |
| 14 | $3(7x - 4) + 2(3x + 1)$ |
| 15 | $3x(2x^2 + 3y)$ |

**Consolidation Extension Questions:**

| | |
|---|---|
| 16 | $12x^2y^8 \div 4xy^5$ |
| 17 | $\dfrac{\left(3x^2y\right)^2}{(xy)^2}$ |
| 18 | $\dfrac{1}{3}(3x+6)+\dfrac{1}{4}(4x-12)$ |
| 19 | $-7x^5 \cdot \left(2x^3\right)^2$ |
| 20 | $2\,[3+2\,(x-6)]+3\,[-2\,(x-5)+8]$ |
| 21 | $-\dfrac{1}{4}(4x-3y)-\dfrac{3}{5}(6x-10y)$ |
| 22 | $0.2x\,(x-5)+0.4x\,(3x-2)$ |
| 23 | $\dfrac{4x^4y^3 \cdot 6xy^4}{3x^3y \cdot 8xy^2}$ |
| 24 | $4x\,(x-3)-2\left(x^2-3x+4\right)-\left(x^2-5\right)$ |
| 25 | $\dfrac{\left(3xy^2\right)^3 \cdot \left(-8x^2y\right)}{\left(2x^2y^2\right)^2}$ |

**Consolidation Combination Questions Version 2:**

| | |
|---|---|
| 1 | $x^4 y^2 \cdot xy^2$ |
| 2 | $4x + 5x^2 - 2x^2 + 8x$ |
| 3 | $5(3x - 7)$ |
| 4 | $x^4 \cdot x^6$ |
| 5 | $\left(y^4\right)^5$ |
| 6 | $-6(-2x + 4)$ |
| 7 | $x^6 y^7 \div x^3 y^2$ |
| 8 | $5xy + 3x^2 - 2x^2 y - 5x + x^2 - 7y$ |
| 9 | $4x(5x + 2) + 2x(3x + 4)$ |
| 10 | $y^8 \div y^3$ |
| 11 | $x^3 y \cdot x^2 y^5$ |
| 12 | $-5x(3x + 4)$ |
| 13 | $\left(x^4 y\right)^3$ |
| 14 | $4(6x - 3) + 5(2x + 1)$ |
| 15 | $2x\left(4x + y^2\right)$ |

**Exit Card Sample:**

Name: ___________________________

Today we investigated 6 different algebraic properties. In each of the following boxes, create a question and then answer it to show me what you know about these properties.

| **Multiplying Powers with the Same Base** | **Dividing Powers with the Same Base** |
|---|---|
| **Power of a Power** | **Collecting Like Terms** |
| **Distributive Property with Numbers** | **Distributive Property with Variables** |

**Python Text Coding Examples:**
Code font is most appropriate for spacing.
Highlight indicates a line in the code that was adjusted from the original.
Please note all lines beginning with a # are comments not code. By putting a # in front of a line, it is "commented out" so that it is not considered code.

The first version of the code shown here can be used as is with TI-Nspire CX handhelds or the student or teacher premium software.

The second set of code can be used within Python or be run using a compiler such as Programiz.

Alternatively, the original file "Multiplying Powers with the Same Base" can be found by clicking on this link Exponents-Revised. You can sign up for Replit by using a Google account, or by creating a Replit account. To modify this code, you MUST make a copy of the file. To do this, click the "Fork" button. You will be able to see the code, change it, and then "run" it to test the changes.

**Code for TI**

**Exponents Coding Setup: (This must be called "exponents" in order to work.)**

```
#To load in TI first….
from math import *

def expn(val):
  my_exponent = ""
  if (val < 0):
   my_exponent = "\u2212"
  # We have dealt with the sign now take the absolute value of the
exponent and convert it to a string
  val = abs(val)
  # Convert it to a string to read one character at a time
  val = str(val)
  for d in val:
    if (d == "0"):
      my_exponent = my_exponent + "\u2070"
    elif (d == "1"):
      my_exponent = my_exponent + "\u00b9"
    elif (d == "2"):
      my_exponent = my_exponent + "\u00b2"
    elif (d == "3"):
      my_exponent = my_exponent + "\u00b3"
```

```
    elif (d == "4"):
      my_exponent = my_exponent + "\u2074"
    elif (d == "5"):
      my_exponent = my_exponent + "\u2075"
    elif (d == "6"):
      my_exponent = my_exponent + "\u2076"
    elif (d == "7"):
      my_exponent = my_exponent + "\u2077"
    elif (d == "8"):
      my_exponent = my_exponent + "\u2078"
    elif (d == "9"):
      my_exponent = my_exponent + "\u2079"
    else:
      my_exponent = my_exponent + ""
  return my_exponent
```

**Multiplying Powers with the Same Base (original code)**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents.
from math import *
from exponents import *

# Collect the base value
base = input("What is the common base? ")
# Incorporate the user base value into the prompt for the first
exponent
prompt = "For the 1st factor, "+base+" is raised to what power?
"
# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into the prompt for the second
exponent
prompt = "For the 2nd factor, "+base+" is raised to what power?
"
# Collect the second exponent
secondpower = input(prompt)

# Add the exponents
powersum = int(firstpower) + int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
```

```
    result = result+" · "

# Store the original problem
problem = base + expn(int(firstpower)) + " · " + base +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```

**Dividing Powers with the Same Base:**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents
from math import *
from exponents import *

# Collect the base value
base = input("What is the common base? ")
# Incorporate the user base value into
# the prompt for the first exponent
prompt = "For the numerator, "+base+" is raised to what power? "
# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into
# the prompt for the second exponent
prompt = "For the denominator, "+base+" is raised to what power?
"
# Collect the second exponent
secondpower = input(prompt)

# Subtract the exponents
powersum = int(firstpower) - int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
    result = result+" · "

# Store the original problem
problem = base + expn(int(firstpower)) + " / " + base +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```

**Power of a Power**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents
from math import *
from exponents import *

# Collect the base value
base = input("What is the base? ")
# Incorporate the user base value into
# the prompt for the first exponent
prompt = "The "+base+" is raised to what power? "

# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into
# the prompt for the second exponent
prompt = "("+base+")"+expn(int(firstpower))+" is raised to what
power? "
# Collect the second exponent
secondpower = input(prompt)

# Multiply the exponents
powersum = int(firstpower) * int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
    result = result+" · "

# Store the original problem
problem = "(" + base + expn(int(firstpower)) + ")" +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```

Code for use with an online Python compiler such as: Programiz or Replit.

**Multiplying Powers with the Same Base (original code)**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents
from math import *

#print("x\u00b2 + y\u00b2 = 2")  # x² + y² = 2

def expn(val):
  my_exponent = ""
  if (val < 0):
   my_exponent = "\u2212"
  # We have dealt with the sign now take the absolute value of
the exponent and convert it to a string
  val = abs(val)
  # Convert it to a string to read one character at a time
  val = str(val)
  for d in val:
    if (d == "0"):
      my_exponent = my_exponent + "\u2070"
    elif (d == "1"):
      my_exponent = my_exponent + "\u00b9"
    elif (d == "2"):
      my_exponent = my_exponent + "\u00b2"
    elif (d == "3"):
      my_exponent = my_exponent + "\u00b3"
    elif (d == "4"):
      my_exponent = my_exponent + "\u2074"
    elif (d == "5"):
      my_exponent = my_exponent + "\u2075"
    elif (d == "6"):
      my_exponent = my_exponent + "\u2076"
    elif (d == "7"):
      my_exponent = my_exponent + "\u2077"
    elif (d == "8"):
      my_exponent = my_exponent + "\u2078"
    elif (d == "9"):
      my_exponent = my_exponent + "\u2079"
    else:
```

```
      my_exponent = my_exponent + ""
  return my_exponent

# Collect the base value
base = input("What is the common base? ")
# Incorporate the user base value into the prompt for the first
exponent
prompt = "For the first factor, "+base+" is raised to what
power? "
# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into the prompt for the second
exponent
prompt = "For the second factor, "+base+" is raised to what
power? "
# Collect the second exponent
secondpower = input(prompt)

# Add the exponents
powersum = int(firstpower) + int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
    result = result+" · "

# Store the original problem
problem = base + expn(int(firstpower)) + " · " + base +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```

**Dividing Powers with the Same Base:**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents.

from math import *
#print("x\u00b2 + y\u00b2 = 2")  # x² + y² = 2
```

```
def expn(val):
  my_exponent = ""
  if (val < 0):
   my_exponent = "\u2212"
  # We have dealt with the sign now take the absolute value of 
the exponent and convert it to a string
  val = abs(val)
  # Convert it to a string to read one character at a time
  val = str(val)
  for d in val:
    if (d == "0"):
      my_exponent = my_exponent + "\u2070"
    elif (d == "1"):
      my_exponent = my_exponent + "\u00b9"
    elif (d == "2"):
      my_exponent = my_exponent + "\u00b2"
    elif (d == "3"):
      my_exponent = my_exponent + "\u00b3"
    elif (d == "4"):
      my_exponent = my_exponent + "\u2074"
    elif (d == "5"):
      my_exponent = my_exponent + "\u2075"
    elif (d == "6"):
      my_exponent = my_exponent + "\u2076"
    elif (d == "7"):
      my_exponent = my_exponent + "\u2077"
    elif (d == "8"):
      my_exponent = my_exponent + "\u2078"
    elif (d == "9"):
      my_exponent = my_exponent + "\u2079"
    else:
      my_exponent = my_exponent + ""
  return my_exponent



# Collect the base value
base = input("What is the common base? ")
# Incorporate the user base value into
# the prompt for the first exponent
prompt = "For the numerator, "+base+" is raised to what power? "
# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into
```

```
# the prompt for the second exponent
prompt = "For the denominator, "+base+" is raised to what power?
"
# Collect the second exponent
secondpower = input(prompt)

# Subtract the exponents
powersum = int(firstpower) - int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
    result = result+" · "

# Store the original problem
problem = base + expn(int(firstpower)) + " / " + base +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```

**Power of a Power**

```
# This will work for user input non-numeric (literal) bases and
#integer exponents.

from math import *

def expn(val):
  my_exponent = ""
  if (val < 0):
   my_exponent = "\u2212"
  # We have dealt with the sign now take the absolute value of
the exponent and convert it to a string
  val = abs(val)
  # Convert it to a string to read one character at a time
  val = str(val)
  for d in val:
    if (d == "0"):
```

```
      my_exponent = my_exponent + "\u2070"
    elif (d == "1"):
      my_exponent = my_exponent + "\u00b9"
    elif (d == "2"):
      my_exponent = my_exponent + "\u00b2"
    elif (d == "3"):
      my_exponent = my_exponent + "\u00b3"
    elif (d == "4"):
      my_exponent = my_exponent + "\u2074"
    elif (d == "5"):
      my_exponent = my_exponent + "\u2075"
    elif (d == "6"):
      my_exponent = my_exponent + "\u2076"
    elif (d == "7"):
      my_exponent = my_exponent + "\u2077"
    elif (d == "8"):
      my_exponent = my_exponent + "\u2078"
    elif (d == "9"):
      my_exponent = my_exponent + "\u2079"
    else:
      my_exponent = my_exponent + ""
  return my_exponent



# Collect the base value
base = input("What is the base? ")
# Incorporate the user base value into
# the prompt for the first exponent
prompt = "The "+base+" is raised to what power? "
# Collect the first exponent
firstpower = input(prompt)
# Incorporate the user base value into
# the prompt for the second exponent
prompt = "("+base+")"+expn(int(firstpower))+" is raised to what
power? "
# Collect the second exponent
secondpower = input(prompt)

# Add the exponents
powersum = int(firstpower) * int(secondpower)

# Create empty string to collect x bases
result = ""
# Loop through and add base x times
```

```
for i in range(powersum):
  result = result+base
  # If it's not the last one, add multiplication sign
  if (i < powersum-1):
    result = result+" · "

# Store the original problem
problem = "(" + base + expn(int(firstpower)) + ")" +
expn(int(secondpower))
# Store the final solution
solution = str(base+expn(powersum))

# Print the answer
print(problem)
#print(" = ",result)
print(" = ",solution)
```