LESSON DETAILS

Change One Thing

Lesson Summary

This lesson allows students to explore how changing one or more dimensions in a 2D shape affects perimeter/circumference and/or area, and how changing one or more dimensions in a 3D object affects surface area and/or volume. Students will use coding to gain further understanding of these concepts, allowing them to explore a greater breadth of examples during the lesson.

Grade: 9

Big Ideas

The development and use of geometric and measurement relationships. Understanding how changing dimensions affects surface area and volume.

Learning Expectations

AA1. develop and explore a variety of social-emotional learning skills in a context that supports and reflects their learning in connection with the expectations across all other strands.

- building healthy relationships and communicating effectively in mathematics;
- developing critical and creative mathematical thinking.

A1. apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- problem solving
- reasoning and proving
- selecting tools and strategies

A2. make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers.

C1. demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

> **C1.4** simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts.
> **C1.5** create and solve equations for various contexts, and verify their solutions.

C2. apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands

> **C2.1** use coding to demonstrate an understanding of algebraic concepts including variables, parameters, equations, and inequalities
> **C2.2** create code by decomposing situations into computational steps in order to represent mathematical concepts and relationships, and to solve problems

E1. demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations

> **E1.4** show how changing one or more dimensions of a two-dimensional shape and a three-dimensional object affects perimeter/circumference, area, surface area, and volume, using technology when appropriate.

Cross Curricular Connections

Computer science (changing of shapes through coding and graphic design)

Learning Goals and Success Criteria:

LG1: We are learning how area, surface area and volume change as dimensions in 2D shapes and 3D objects change.

SC1: I can calculate perimeter, area, surface area and volume.
SC2: I can describe, in words, how a shape is changing.
SC3: I can create a complete question about a 3D object to determine how the surface area and volume will change when one dimension changes.

LG2: We are learning to code with variables.

SC1: I can code an interactive volume and surface area calculator.
SC2: I can write code to substitute values into a volume or surface area formula to calculate surface area and volume.

CONSIDERATIONS THROUGHOUT THE LESSON

Differentiated Instruction and Universal Design for Learning

Use Visibly Random Groupings to create small groups.
Use flexible small group instruction for students who need support.
Adjust the geometric shapes to avoid frustration/boredom.
Provide diagrams for students who are struggling or who are visual learners.
Provide a variety of manipulatives.
Consider extensions for groups that finish early.
During consolidation, students will choose a 3D object that will allow them to demonstrate their level of understanding.
Scaffolding: teacher prompts are included throughout the lesson.

Assessment

Throughout the lesson, the teacher will be listening for students correctly and effectively using mathematical language to describe their mathematical thinking.
Listen for understanding of how area, surface area and volume change when a dimension is changed in both 2D shapes and 3D objects.
Listen for precise language during the sharing of the Minds-On “Which One Doesn’t Belong?” Activity. Insights into student comfort with surface area and volume may be apparent during the sharing.
Students will self evaluate using the Success Criteria.
Student created questions (see Consolidation section)
Use descriptive feedback during the Action part of the lesson, including the coding section, using the three-part structure – done well, needs improvement, how to improve.
Provide oral feedback to students while they are practising the assigned tasks. Using this structure, provide feedback to individuals, groups, and the whole class.
Peer descriptive feedback in Consolidation

RESOURCES AND LEARNING ENVIRONMENT

Educator Resources Needed

Computer with Internet
Which One Doesn't Belong image
Given 2D and 3D situations

Student Materials Needed

Computer with Internet
Graph paper
Vertical whiteboard (or chart paper if VNPS are not available)
Dry erase markers

Learning Environment Considerations

The Minds-On section starts with students in a whole class Which One Doesn't Belong activity. Students will think individually, then share their individual responses. Consider having all students clustered around the images provided in the Which One Doesn't Belong activity. For students who need the support of a physical handout to help them guide and communicate their thinking, several handouts should be available.

In the Action section, students will work in small groups (2-3 students) selected by the teacher. Each group will need space to work with a vertical non-permanent surface. They should be able to compare ideas with other groups. They will also need access to computers for the coding portion (Part 2).

The Consolidation will be completed as a whole class and in pairs.

Consider providing a vocabulary list aligning perimeter/circumference with surface area and area with volume. If students seem confused by these alignments, have a discussion as to why perimeter (rather than area) "turns into" surface area as we move from shapes to objects, and area connects to volume. Some actual shapes and containers may aid students' understanding of these relationships.

LESSON CONTENT

Minds-On (20-30 Minutes)

Which One Doesn't Belong?

Have all students gather around a projected image of the Which One Doesn't Belong image (below). Ask them to mentally select one of the four objects shown and to come up with one way in which it does not belong with the other three objects in the image. Once they have done that, they should pick another and find a way in which it is different from the other three. They will continue until they have at least one reason why each object is different from the other three. Let the students know that they will be asked to share their thinking and reasoning using clear mathematical language. Reassure them that, for each object, there are many ways in which it could be described as being different from the other three, so this exercise is about thinking, reasoning, and communicating and not about getting “the right answer”. Remind the students that this should be done quietly, until you (the teacher) ask for their responses.

After 3 - 5 minutes of quiet time, collect student responses orally. All students should be encouraged to share their thinking, but honour a students’ decision to “pass”. (You may want to check back after some students have shared. A student may decide to share after they have heard some responses from their peers.)
If students have not included surface area and volume in their responses, you may prompt them to speculate how these may be different among the objects shown.

Sample student responses:
Top left does not belong because it is the only one:

- where the length, width and height are all equal.
- that doesn’t look as if the dimensions are correct.
- that is a cube.

Top right does not belong because it is the only one:

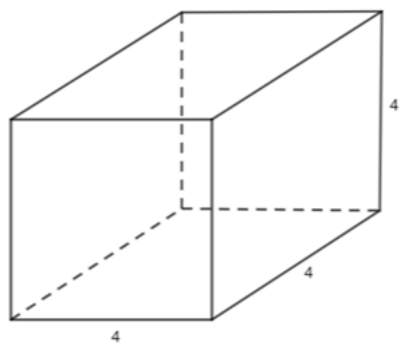
- with non-integer side lengths.
- that has 2 triangular faces.
- that looks like a tent.
- that doesn’t have a square base.

Bottom left does not belong because it is the only one::

- that is not a prism.

Bottom right does not belong because it is the only one:

- with a surface area that is not 96 units^2 (Note: it is not anticipated that students will perform these sorts of calculations during this Minds-On activity.)
- that doesn’t have a height of 4.
- that has a height of 6.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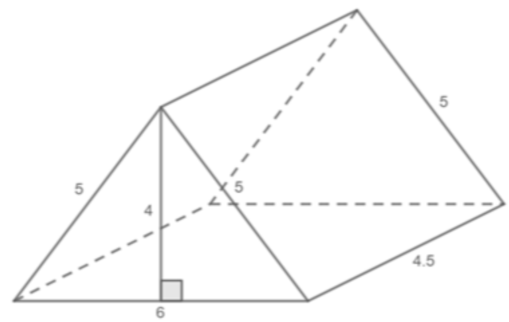
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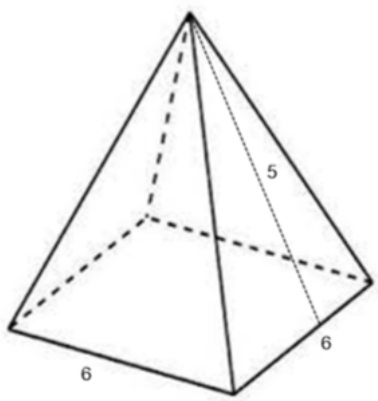
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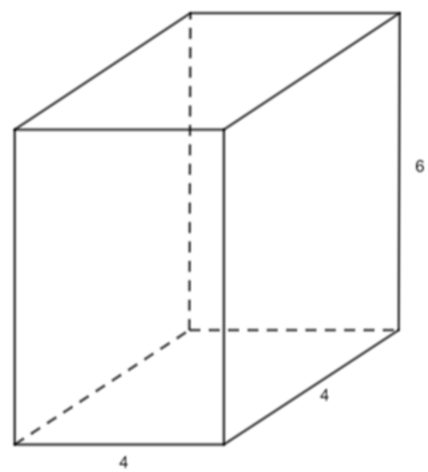
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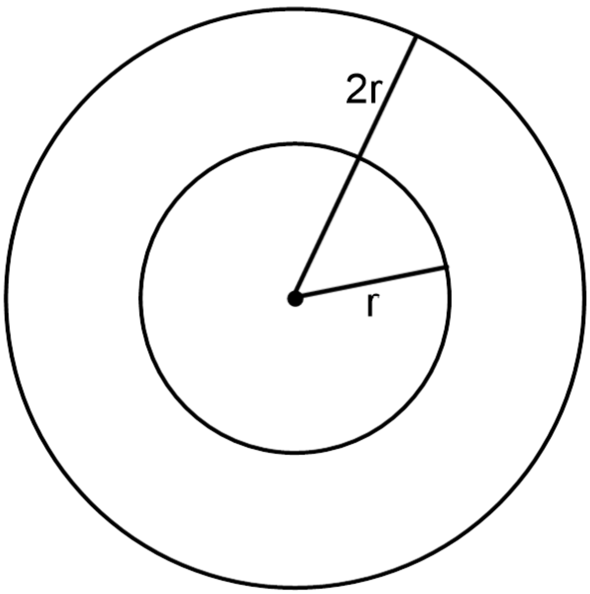
Action (120 minutes)

Part 1

Students will work in small groups on the following two tasks. Each group is presented with the following problems:



A grade 9 class wants to create a circular community garden as a school project. They want to enclose the garden with a small fence. They plan to use a rope to determine how big the garden should be:

They place a wooden stake in the middle of the space and attached the rope to the stake. The length of the rope from stake to end is 4 meters.

a) What length of fencing will they need to enclose the garden?

b) What is the area of the garden this length of fencing would enclose?

Teacher scaffolding prompts: Draw the fence and colour the garden - which is circumference and which is area? Provide formulas for circumference and area of a circle as needed.

For this next question, students will be asked to consider a larger circular area. They will likely perform the same calculations as for the first two questions. The purpose of these questions, though, is to draw out the relationship between the change in radius and the changes in circumference and area. As students work on this new problem, circulate to observe if any groups are making this connection independently. You may see that some groups just require a small prompt to shift their attention from the computation to the relationships.

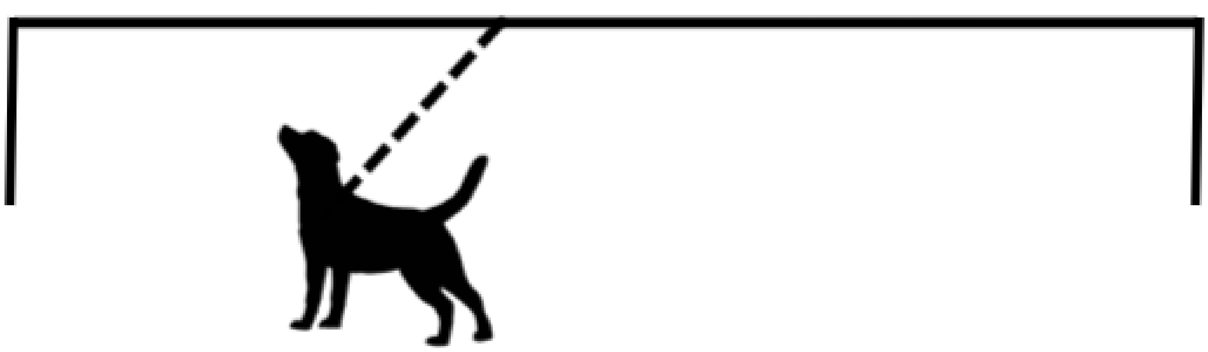
c) If they double the length of the rope, how would the circumference and area change? Teacher scaffolding prompt: Look at the diagram provided - what has changed? Has anything stayed the same?

d) The class isn't sure if they will have enough money to buy the amount of fencing required for the larger garden. If they only fence an enclosed semicircular garden having the larger radius, how much fencing will be needed?

Teacher scaffolding prompts: Draw a diagram of the new garden. How does this new area compare to the previous one?

Students will compare their answers with other groups to look for different strategies.

Students will be using vertical non permanent surfaces if possible for the following question as we will be doing a gallery walk to consolidate.

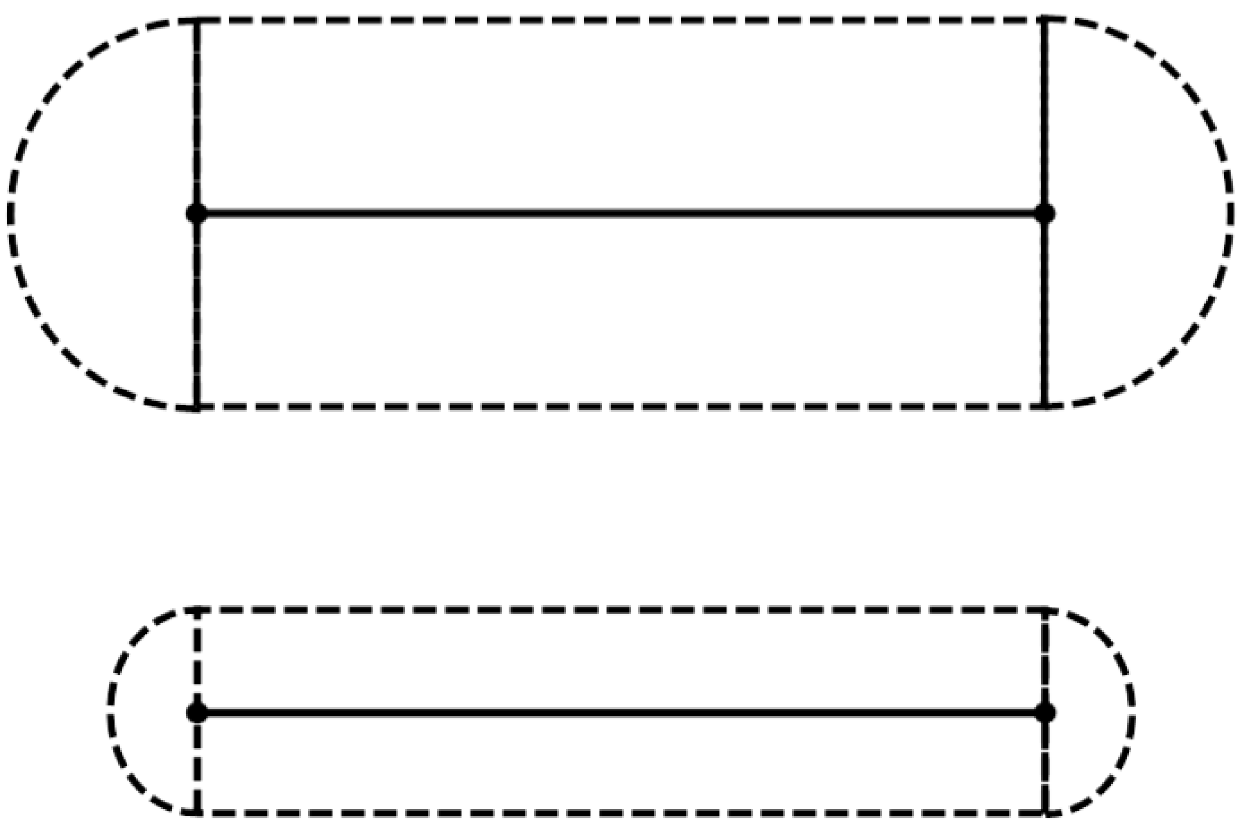


Students will be using vertical non permanent surfaces if possible for the following question as we will be doing a gallery walk to consolidate.

At a campground, Jax, the dog, is tied to a rope attached to two low posts that are 6 m apart. Assuming that his leash stays horizontal, determine the path he would create if he walks as far as possible on both sides of the rope with the leash fully extended.

Determine the effect on how much area Jax has if his leash is shortened to half its original length. Each group should choose their own initial length for the leash and should make their thinking visible.

As the teacher circulates, the diagrams below may be provided to groups that are struggling. Each group should be able to generalize their findings and convince others with mathematical arguments.



During the gallery walk students will look for similarities and differences between the groups.

Summarize the findings as a whole class.

Part 2

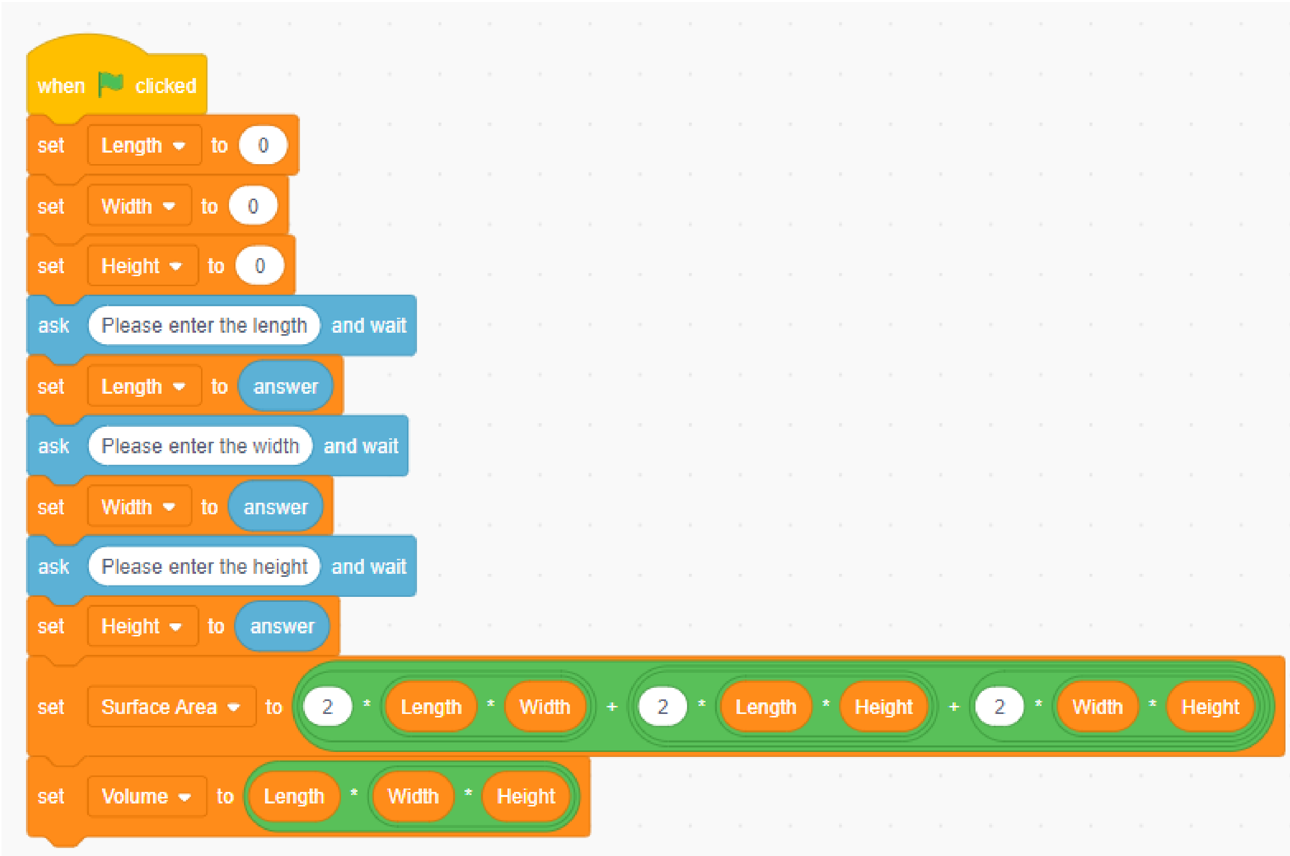
In this part of the Action, students are asked to use code to quickly determine the effects of changing dimensions of an object on the volume and surface area of the object, allowing them more time to focus on the relationships (patterns) they see. Students will create code in Scratch (or another coding platform used in your class) that will ask for the dimensions of a 3D object to be input, and the code will return as output both the surface area and the volume of that object. To accomplish this students will need to:

1. Create a variable for each dimension and for surface area and volume.
2. Set the value of each variable to 0. (Initialize)
3. Prompt the user to input each dimension. This data needs to be stored in the appropriate variable.
4. Calculate surface area and volume.

It is essential that the correct formulas for volume and surface area are used, and that they are input correctly (with appropriate brackets, etc.) The teacher should circulate to verify that these steps are done correctly, or students' conclusions will be invalid. While circulating, the teacher will offer support to students who are struggling. Chunks of code can be shown and explained to students, as necessary, as students progress to help ensure that all students complete this task.

To start, reassign the small groups so that students are working with different partners. Assign a solid to each group for them to investigate through code. Some objects are: a sphere (only allows for changing one dimension), a cube (again, only allows for changing one dimension), a square-based prism (3 dimensions will change but two are the same), 2 or 3 non-cuboid, non square-based rectangular prisms (each having different initial dimensions), a square-based pyramid, a non square-based pyramid, a cylinder, and a cone.

Rectangular prism sample code:



Students can use their code to quickly calculate the surface area and volume as they change one dimension at a time to complete the following prompts.

Possible prompts:

When I double the {insert dimension name here}, the surface area ____________ , and the volume ________.

When I triple the {insert dimension name here}, the surface area ____________ , and the volume ________.

When I halve the {insert dimension name here}, the surface area _____________ , and the volume _______________.

While students are working on their code, teachers will circulate and observe students working in order to identify examples to share with the whole class. Good candidates for sharing might include :

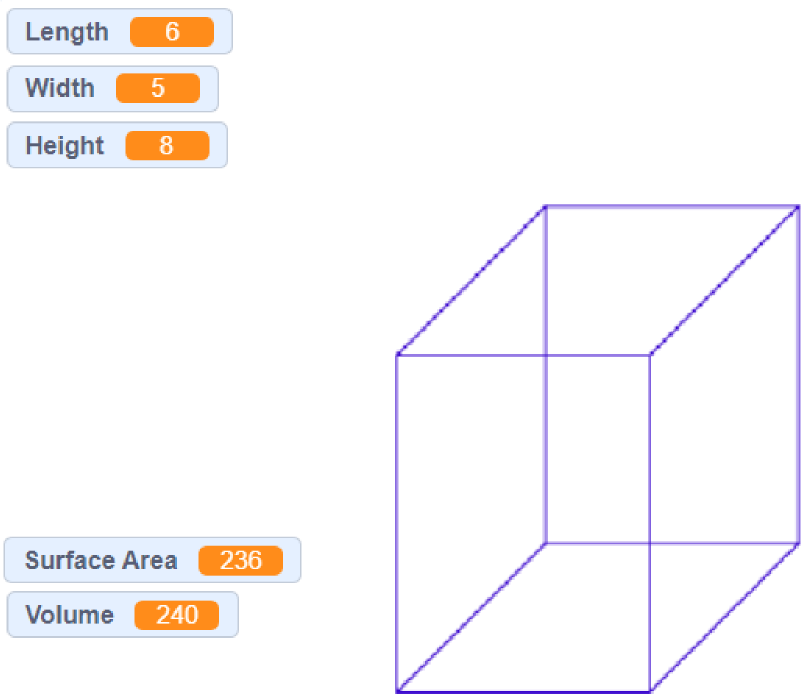
- Efficient code (fewer lines to achieve desired result)
- Creative solutions to the problem

Students who successfully complete their given investigation may want to try another. Students should also be instructed to share the code with the teacher so the teacher has access for the Consolidation step.

Extension Opportunities

How does changing the surface area and volume (increasing or decreasing) affect the dimensions?

Create code to draw the 3D object with dynamic dimensions.



Given a 3D object with initial dimensions provided, determine the new dimensions that will result in the surface area or volume doubling.

Consolidation (20-30 minutes)

As a whole class, students, who are still with their small group partners, will be invited to observe some of the code (chosen by the teacher) that was written at the end of the Action in order to realize that there are different ways to solve the same problems.

Prepare a chart that has 1 row for each object students will have investigated using code. Have a column for “Observations about Volume”, one with “Observations about Surface Area”, and one labelled “Explanation”. Start collecting students’ observations, using one of the objects that has only one variable (sphere, cube). What pattern did students observe? (Listen for students using precise language to describe their observations.) Record these on the chart. Move to objects that have 2 independent variables (such as a cylinder and a square-based prism), then 3 distinct variables. Provide students with access to this observation chart (sharing a slide or a pdf of an interactive whiteboard image, allow students to take a photo, give them time to copy the chart into their notebooks, …) Bringing two groups together, have students discuss any patterns they see. They may want to make note of any new and interesting conclusions they draw. Elicit their thinking, accepting one idea from each group until all ideas have been shared.

Anticipated student observations:

- A sphere and a cube act in a similar way.
- When you have the same object but one has a square base and the other doesn’t, you get very different results. For example, doubling the base length creates a large change in volume compared to doubling the base length when the base is not a square.
- Doubling the height always seemed to double the volume.

Student-Created Questions. Students work in pairs to create a question about changing a dimension in a 3D object. Each student should choose a 3D object (simple or composite) that will allow them to demonstrate their learning. Their question should ask how the surface area and volume will change if one dimension changes. The questions must include all pertinent information including measurements and how the dimension has changed. They will then give the question to their partner who will attempt to solve it. If the partner feels there is missing information, they will give feedback. This exchange continues until the problem can be solved.

The teacher will collect the questions and use them to check for student understanding.