LESSON DETAILS

How Many Equations Can a Line Possibly Have?

Lesson Summary

Students will make connections between algebraic models of linear equations and their representations (graph/table of values/pattern), and describe the effect of the representations when an equation is manipulated in different ways.

Grade: 9

Big Ideas

Algebraic equivalence, Representations of linear relationships, Equivalent Linear Relationships, Transformations,Translations, Rotations, Reflections

Learning Expectations

AA1. develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands

- Building healthy relationships and communicating effectively in mathematics
- Identifying resources and supports that aid perseverance in mathematical learning
- Developing critical and creative mathematical thinking

A1. apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- Connecting
- Representing
- Communicating

A2. make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

C1. demonstrate an understanding of the development and use of algebraic concepts and their connection to numbers, using various tools and representations

C1.3 compare algebraic expressions using concrete, numerical, graphical, and algebraic methods to identify those that are equivalent, and justify their choices
C1.4 simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts

C4. demonstrate an understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate
C4.1 compare characteristics of graphs, tables of values, and equations of linear and non-linear relations
C4.2 graph relations represented as algebraic equations of the form x = k, y = k, x + y = k, x - y = k, ax + by = k, and xy = k, and their associated inequalities, where a, b, and k are constants, to identify various characteristics and the points and/or regions defined by these equations and inequalities
C4.3 translate, reflect, and rotate lines defined by y = ax, where a is a constant, and describe how each transformation affects the graphs and equations of the defined lines

Cross Curricular Connections

Learning Goals and Success Criteria:

These are some suggested learning goals for this lesson. Ideally, the success criteria should be co-created with students when beginning the lesson.

LG1: We are learning that the graph/table/pattern of one particular line can be represented using many different-looking equations.
SC1: I can write a given equation of a line in different ways
SC2: I can provide mathematical evidence that two different-looking equations of lines have the same graph, table of values, and geometric pattern.

LG2: We are learning to connect a transformation of a line's graph and the corresponding change to the line's equation.
SC1: I can describe the transformation of the line's graph using mathematical vocabulary (translation, rotation, reflection).
SC2: I can describe the transformation that will be made to the graph of a line when its equation is changed.
SC3: I can transform the line defined by the equation y = ax to rotate, translate, or reflect the line.

CONSIDERATIONS THROUGHOUT THE LESSON

Differentiated Instruction and Universal Design for Learning

Consider the following when planning instruction based on student strengths and needs in the class:

- **Use a variety of materials that represent all modalities**: Example materials include manipulatives to construct patterns and make connections to number and algebra (examples include integer chips, algebra tiles, blocks), use of graphing technology, and use of visual representations.
- **Present information using multi-modal approaches**: Pair visual and auditory formats or kinesthetic and auditory formats when explaining concepts, definitions, or instructions.
- **Scaffolding**: Students may need varied support to complete all parts of the lesson. This can include breaking the investigations into smaller steps, rephrasing or re-explaining parts of the lesson, explaining concepts in multiple ways, and extending time limits.
- **Assistive Technology*:*** Provide students with access to assistive technology softwares like text-to-speech and speech-to-text to support students with reading comprehension and written expression.
- **Provide Choice**: Allow students to choose learning materials and ways to present their thinking.
- **Small Group Instruction**: Consider small-group instruction sessions for students who may need remedial support in algebraic simplification (combining like terms, use of distributive property, and balance equations) and linear properties (rate/slope, initial value/y-intercept, and equations of lines)
- **Preview Vocabulary**: Provide students with a list of vocabulary for this lesson to help support comprehension. The list of vocabulary words/phrases could include equations, linear relations, equivalent linear equations, original and image lines, base graph, transformation, translation, reflection, rotation.

Assessment

Observation Opportunities

While students are collaborating and investigating, circulate and note evidence of students showing connections between algebraic understandings and representations of lines and the effect of changes to an algebraic representation on the graph of a line. If you notice any students who are struggling to make these connections, intervene and use questioning or other differentiated supports to lead them in making valuable connections. Offer descriptive feedback on application of algebraic skills, effective reflection during the problem solving process, managing stress and persevering, and use of mathematical terminology when communicating.

Conversation Opportunities
While circulating, ask students to explain their thinking process to assess attainment of the learning goals. All product opportunities can be adapted into an interview process to accommodate students who need or prefer oral responses.

Checkpoints
There are two checkpoints during the Action at which time the teacher will assess student understanding of the learning in the preceding section.

Product Opportunities
There are multiple products in this lesson that teachers can use to assess that students are meeting learning goals. Teachers can choose to use these as additional assessment checkpoints during the lesson. These can be shared with students as paper handouts or digital files (this will allow students to access assistive technologies).

- Creating Equivalent Equations of Lines - this product provides evidence of LG1 (both success criteria).
- Which Equation Is It? - Students will be generating equations that their peers can use as practice questions. The practice questions can be collected to provide students with descriptive feedback.
- New Equations - New Graphs - students are recording their thinking during a learning activity. Teachers can use this to assess LG2.
- Putting It All Together - The focus of this page is to demonstrate a conceptual understanding of the connection between the algebraic representation and the other representations of a linear relationship.

RESOURCES AND LEARNING ENVIRONMENT

Educator Resources Needed

Projector
Markers, Poster Paper (if non-permanent vertical space is not available)
Animated images (for Part 3 of Action)
Access to graphing technology and other digital supports

Student Materials Needed

Paper copies or digital access to the following documents:

- Creating Equivalent Equations of LInes
- Investigation Transformations of Linear Relationships
- New Equations - New Graphs
- Putting It All Together
- Which Equation Is It?

Learning Environment Considerations

This activity will work well in small groups of 2-3 students using a flexible grouping strategy. Groups can be changed throughout the lesson to encourage class-wide collaboration. When creating groups, consider the following strategies:

Mixed-ability groupings can be created with students who are comfortable using different representations (concrete representations, graphical/numerical representations, and algebraic representations). Teachers can then observe and prompt students in each group to explore the learning goals and success criteria using multiple representations so that students can learn from each other.

Like-ability groupings can be created with students so that teachers can create differentiated tasks and targeted interventions. For example, teachers can group students who need support in making connections between concrete representations of a linear pattern and graphical representations (and provide the students with manipulatives and graphing technology). Introducing a component of the lesson for students to see a connection between the two representations can enhance the learning of students in this group.

The classroom should be set up so that students can work together in small groups and have a place to record and demonstrate their thinking. Vertical, non-permanent surfaces can allow students to do this (other vertical options, like poster paper, can also work).

LESSON CONTENT

Minds-On (25 Minutes)

Divide students into groups of 2-3 using a flexible grouping strategy (see Learning Environment Considerations). Remind students of co-constructed norms for group work (or point them out to students if they are posted in the classroom) and that they are working together to meet the learning goals of this activity.

Consider the equation y = -2x + 5. Ask students to work in their groups to represent the linear relation using: manipulatives/diagrams, a table of values, and a graph (or use the provided image). Ask students the following questions:

- How are the representations similar? How are they different?
- How does each representation (pattern diagram, table of values, and graph) model the same relation?

This part will provide an opportunity to activate prior knowledge of linear relationships.

Equation: $y = -2x + 5$

Pattern:

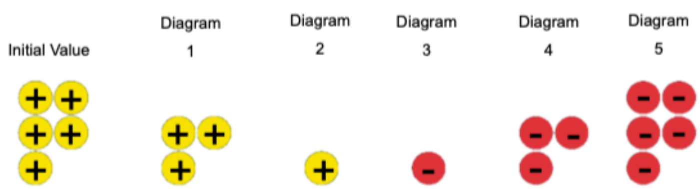
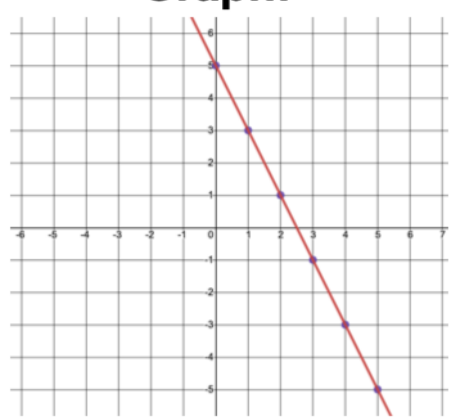


Table of Values:

x_1	y_1
0	5
1	3
2	1
3	−1
4	−3
5	−5

Graph:



(Table of Values and Graph generated using [Desmos | Graphing Calculator])

When students are finished, carry out a short class discussion that addresses the questions above. Have students explain any connections between the representations using the visuals they've created or the visual you provided.

Action (90 Minutes)

Introduce and explain LG1 and co-construct the success criteria so that students are aware of how the learning goal is achieved. Provide students with an outline of the three parts of this lesson (timelines for each part and what to expect each day). Discuss strategies to persevere when it might be difficult to make a connection in the problem solving process (brainstorm ideas with your partner, ask questions, take a quick break and come back to it, reflect on the learning goals, positive self-talk) and reinforce beliefs that students are capable learners and will be able to meet the learning goals.

Part 1 - Creating Equivalent Equations

Provide students with a paper copy or digital access to the Creating Equivalent Equations of Lines handout. Ask students to use what they know about numbers, patterns and algebra to create a new equation that has the same graph/table/pattern as each of the equations given on their sheet. When they've created their equation, they will need to use another representation of the linear relationship (graph/table of values/geometric pattern) to show that it is equivalent. They should know that they will be asked to explain their process as well as their reasoning about equivalence. (Ex: Students might use a graphing calculator or compare the points in a table of values or draw the two geometric patterns to check for equivalence.)

Of the three equations asked, encourage students to create one equation that looks like it is obviously equivalent to the original (but looks different), one that looks quite different, and one that looks completely different and you would never guess it was equivalent to the original. (Educator Prompt: Students can create more complicated equations by continuously adding or subtracting multiple terms on both sides or multiplying/dividing by numbers on both sides. This gives students an opportunity to make equivalent linear relationships and get mathematically creative.) See Sample Response.

Ensure you are circulating throughout the room to offer support and prompt students when needed. When they have completed this task, ask them to show you their sheet and offer quick descriptive feedback on their algebraic work and mathematical communication.

Provide students digital access to the Google Doc Which Equation Is It? Instruct students to input their new equations from the Creating Equivalent Equations of Lines handout into any row on the table that is free (table may need to be adjusted depending on number of groups and students in your class). The equations entered will be used as deliberate practice in the consolidation later.

Checkpoint: Use this opportunity to ensure that students are comfortable with the terms "equivalent equations" and "equivalent linear relationships." How might they define these two terms? How might they verify that two equations are equivalent?

Part 2 - Investigating Transformations of Linear Relationships

Provide students with a paper copy or digital access of the Investigating Transformations of Linear Relationships handout.

Inform the students that they are going to transform the given equation into a new equation by adding or subtracting terms or multiplying or dividing factors within the equation. Some of the transformations will change the graphical representation and some of the transformations will not.

When will adding, subtracting, multiplying, and dividing within an equation change the equation?
When will the equation remain unchanged? (Although it may look different, it will be equivalent to the original.)

Demonstrate the use of the "Predict/Observe/Explain" method used in the template and explain how it can be used as a helpful tool when investigating mathematical properties. Students can use the handout as a guide to record their thinking or as a guide to help them show their thinking when presenting to the class. (Educator Prompt: While students are working on this activity in their groups, observe students to see how they make connections between equivalent linear relationships - have conversations with students to lead them in making those connections).

When students have completed the investigation, begin a whole class discussion. Discuss ways to alter the "look" of the equation of a line but keep the graph/table/pattern the same. Describe how "equivalent equations" may not look the "same". Let students share their findings from the investigation and emphasize connections between numerical and algebraic manipulation and the different representations using student work. Also, have the students reflect on the Predict/Observe/Explain strategy and how it helped them describe their thinking and their process as they were creating transformations of, or equivalent representations, of a given line.

Ask students the following. Discuss and agree as a class:

- Can multiple changes be applied to an equation without changing the graph/table of values/pattern?
- How many different algebraic representations exist for a given linear relationship?

- How many equations exist for a given linear relationship?

  Note to teacher: Regarding these last two questions, we can represent a line in an infinite number of ways - for example, we can create equivalent equations by rewriting the variables and constants in different ways. However, each line has a unique equation, even though that equation can be written in different ways and forms.

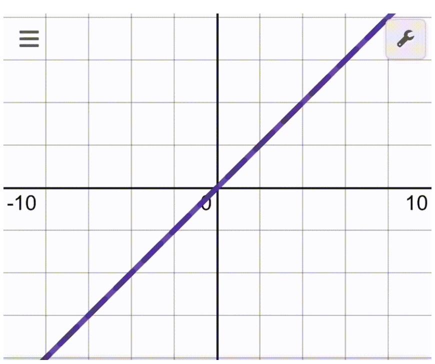
Checkpoint: Use this opportunity to identify and define the 3 types of transformations students have been exploring (translation, rotation, reflection). Ask how transformations of linear relationships are different from equivalent linear relationships.

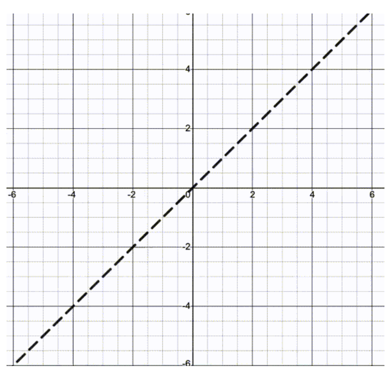
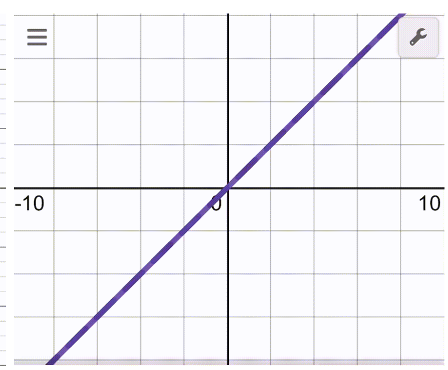
Part 3 - Changes to Equations that also change the graph

For this part of the lesson, students will explore changes to lines that created a change in the other representations. To relate this to geometric terminology, the connection between a line's equation and its graphical representation will be the focus.

As a whole class, review definitions of original line, image line, translation, rotation, and reflection on a Cartesian plane.

Show students the animated images below and ask them to identify if translations, rotations, or reflections are occurring. This will get them to connect the idea of translations, rotations, and reflections to straight lines on a graph.
(Educator Prompt: Students may be more familiar with translation, rotation, and reflections of shapes on a Cartesian Plane).



Explain that translations, rotations, and reflections are terms used to describe visible changes that occur to graphs of lines. We also know from the previous investigation that a graph will not change if we change a line's equation in a "balanced" way.

Could we predict whether a translation, rotation, or reflection occurs by looking at the change in the equation?

Recall LG1 and the related success criteria. Check that students are able to determine what has been achieved and what remains to be achieved during the remainder of the lesson. Introduce and explain LG2 and co-construct the success criteria so that students are aware of how the learning goal is achieved. This can also guide the purpose of the investigation.

In flexible groups of 2-3 students, provide students with the New Equations - New Graphs handout in paper or digital formats. Instruct students to use the Predict/Observe/Explain model to investigate the connection between a change in a linear relationship and the change in that relationship's line graph (whether a translation, rotation, or reflection occurs). Tell students to use graphing technology to check the prediction they made so that they can explain their observations and arrive at a meaningful conclusion. They can use the information on the handout to create a visual summary of their findings to share with the class (on vertical non-permanent surfaces, poster paper, presentation software, etc…).

While students are working through this investigation, observe the ways they are making connections between the algebraic changes and the graphical changes in the line when the line is written in $y = ax + b$ form. Ask students questions and provide descriptive feedback so that students focus on those connections. (Changes in the rate of change in the linear relationship (the slope of the line) are related to rotations/reflections that occur when the "ax" term is multiplied by a constant factor. Similarly, changes to the constant term (the "b" term) in the line's equation are related to translations.) Also, use this opportunity to encourage students to use appropriate vocabulary when describing the graphical changes, use the Predict/Observe/Explain strategy effectively, and work cooperatively to meet the goal of the activity.

Extension Opportunities

- Have students investigate applying other operators that they know to each side of an equation (such as squaring both sides of the equation or taking the square root of both sides of the equation). Do these preserve the linear relationship of the original equation, creating equivalent equations, or do these create new relationships?
- Ask students to explore how reflecting lines through the line $y = 0$ or $y = x$ are the same/different? They can also explore the connection between rotation and reflection of lines - under what circumstances do these coincide? (Note: The animations may assist in making this connection.)

- Encourage students to create equivalent linear equations by applying a variety of operations with unknown constants instead of known constants. Emphasize the beauty of verifying that a complicated equation can be shown to be equivalent to a very simple equation. (Ex: changing $y = 3x + 2$ to $y = 3x + 2 + c$ translates the original line up 3 units if $c > 0$, down 3 units if $c < 0$, and does not create a translation if $c = 0$).
- Ask students to consider what the line $y = x + 1$ would look like if it were translated right 2 units. What would the equation of that new line be? What do you notice? (Translating this line right 2 units creates the same line as translating the original line down 2 units.) Now consider the line defined by $y = 3x + 6$, What would it look like if it were translated right 2 units? What would the equation of that new line be? What do you notice? (Translating right 2 units creates an equation that is equivalent to the equation of the line that has been translated down 6 units.) What pattern or relationship do you see in these two examples?

Consolidation (35 Minutes)

Bring students together for a whole class discussion on their findings from the Part 3 activity or let them do a gallery walk to examine other ways of thinking and communicate their results to their peers. Encourage the students to come to an agreement on the changes that create translations, rotations, and reflections.

During the whole class discussion or gallery walk, ask questions that emphasize the connection between the properties of the line, graphical changes, and algebraic changes. Sample questions could include:

- If I change the slope of the line (rate of change of the relationship), could I create a translation? A reflection? A rotation? When would more than one of these take place?
- What property of the linear relationship or line would you need to change (rate of change/slope or initial value/y-intercept) to translate the line?
- Adding 5 to only the right side of an equation creates a new equation whose graph is translated up 5 units. What if 5 was added to both sides of the equation? How would this be different?

Reintroduce the learning goals and success criteria that were co-constructed earlier and collaboratively refine the success criteria so that the findings from this lesson's investigations are included and the criteria is set for future demonstrations of these learning goals.

Provide students with the Putting it all Together exit ticket as a paper copy or in digital format and ask them to fill it out. Use this as a way to provide students with descriptive feedback and plan next instructional steps (students can submit this by the end of class or, if they need more time, collect it the next day).

Provide students with the list of equations they created in Which Equation Is it? Instruct them to choose any many as they would like and use them as deliberate practice to consolidate the conceptual understanding from today's lessons and develop the corresponding procedural fluency with their algebra skills.

Creating Equivalent Equations of Lines

For each of the equations below, determine an equivalent equation using algebraic techniques. Remember that this means that the graph/table of values/pattern will be the same for both equations.
Then, check that your equation is equivalent by comparing it with another representation.

Equation	Show how you created new equation here	Check that equations are equivalent (explain how you checked below)
$y = x + 1$		
$y = 3x – 7$		
$y = -\frac{1}{2}x + 5$		

Investigating Transformations of Linear Relationships

Transform the linear relationship given by the equation $y = -2x$ by applying the changes below. Then predict if the graph/table/pattern changes. Observe if the change to the equation changed any of the other representations and explain.

Description of Change	New Equation	Predict	Observe: What stays the same? What changes?	Explain
Change the y-intercept from 0 to 1	y = -2x + 1	I predict the graph, table, and numbers in the pattern will change	I graphed the new equation and the graph of the line changed but the new line had the same steepness and still went down to the right	When 1 was added to the y-intercept, it increased the initial value of the line by 1, and also all the other y values by 1, so the line moved up by 1 unit.
Multiply the slope by 2	y = -4x			
Multiply the slope by -1				
Multiply both sides of the equation by 2				
Make up a few more changes to investigate other transformations				

Summary

Describe how somebody could change a line’s equation (in general) and transform the graph.
Describe how somebody could appear to change a line’s equation (in general) but keep the graph the same.

New Equations - New Graphs

Each equation below has been transformed. We know the graph will undergo a transformation as well. Predict if a translation, rotation, or reflection occurs (some may have multiple changes), observe if your prediction is correct by checking on graphing technology, and briefly explain any connections you see. By the end of the activity, you should be able to describe changes that will create translations, rotations, and reflections.

Equation	Change	Predict	Observe	Explain
y = 2x	Change the slope to 6	Translation	A rotation occurred	I thought I would get 6x by adding 4x to 2x, and adding something to the equation would make the line higher, but it didn't. I realize that I can also get 6x by multiplying 2x by 3. That created the rotation.
y = -3x	Change the y-intercept to 5			
y = 6x	Change the slope to 3			
y = ¾x	Multiply the slope by -1			
y = 3x – 5	Change the y-intercept to +5			
Choose your own equation y =	Translate it down 6 units			

y = -2x	Reflect it in the line y = 0			
y = 3x – 1	Write an equivalent equation			
Choose your own equation	Reflect it in the line y = 0, then translate it up 2 units			
Continue to test equations and changes if you need more evidence				

If you think you've seen enough examples and can predict the types of changes that will create translations, rotations, and reflections, summarize your findings in a visual way to present to your peers. Your teacher may ask you to contribute your ideas.

If you need help organizing your ideas, use the template below.

Fill in as many details as possible:

<table>
<tr><td colspan="2">How can you create a translation of an original graph by changing the original equation?</td></tr>
<tr><td>Type of Change and Explanation:</td><td>Visual Example (Graph and Equation):</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">How can you create a rotation of an original graph by changing the original equation?</td></tr>
<tr><td>Type of Change and Explanation:</td><td>Visual Example (Graph and Equation):</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">How can you create a reflection of an original graph by changing the original equation?</td></tr>
<tr><td>Type of Change and Explanation:</td><td>Visual Example (Graph and Equation):</td></tr>
<tr><td></td><td></td></tr>
</table>

Putting It All Together

To reflect on today's activity, answer the following questions below.

1. Describe how you can alter an equation, but not change the graph/table/pattern of a linear relationship. Show an example as part of your explanation.
2. Transform the equation y = 5x so that each graphical change listed below occurs. Explain your process and thinking. Write the equation of the new line.
3. Write the equation of any line. Then, transform it so that the image line is far below the original line. Explain why you changed your equation this way.
4. Sophie Germain, a famous French mathematician, physicist, and philosopher once said "Algebra is nothing more than geometry, in words; geometry is nothing more than algebra, in pictures." Explain what she means using what you've learned so far.

Rotation	**Translation**	**Reflection**

Equation:

Practice: Open the file **Which Equations Is It** and follow Part 2 instructions

Which Equation Is It?

Part 1: Enter your equations from **Creating Equivalent Linear Equations** anywhere in a free row in the table below. One equation per row please.

Part 2: Each of these equations is equivalent to one of $y = x + 1$, $y = 3x – 7$, or $y = -½x + 5$. Which one is it? Show evidence to justify your choice.

#	**Equation**
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Sample responses to Creating Equivalent Equations of Lines

<table>
<tr><th>Equation</th><th>Show how you created an equivalent equation here</th><th>Check that equations are equivalent (explain how you checked below)</th></tr>
<tr><td>$y = x + 1$</td><td>Obvious: $y = 1 + x$

A little less obvious:
$\frac{10 + 5x - 5}{5} = y$

Needs confirmation: $5y - 2x - 8 = 3x - 3$</td><td>Geometric patterns are identical for each of the first 4 figures

Graphs of the original equation and my new equation are the same.

Table of values shows the points are the same for 5 values of x.
<table><tr><td>x</td><td>y = x+1</td><td>New</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>-1</td><td>0</td><td>0</td></tr><tr><td>4</td><td>5</td><td>5</td></tr><tr><td>10</td><td>11</td><td>11</td></tr></table></td></tr>
</table>