LESSON DETAILS

How's It Growing? - Using Visual Patterns to Explore Algebraic Relationships

Lesson Summary

This lesson allows students to see how the way visual patterns are growing can be interpreted in different ways and how each different way connects to an algebraic representation. They will then learn how to show that all the representations are equivalent.

Grade: 9

Big Ideas

Patterns and simplifying algebraic expressions

Learning Expectations

AA1. develop and explore a variety of social-emotional learning skills in a context that supports and reflects their learning in connection with the expectations across all other strands

- building healthy relationships and communicating effectively in mathematics;
- developing critical and creative mathematical thinking skills

A1. apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- Connecting
- Reasoning and Proving
- Representing

A2. make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

C1. demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations

C1.2 create algebraic expressions to generalize relationships expressed in words, numbers, and visual representations, in various contexts

C1.3 compare algebraic expressions using concrete, numerical, graphical, and algebraic methods to identify those that are equivalent, and justify their choices
C1.4 simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts
C1.5 create and solve equations for various contexts, and verify their solutions.

E1. demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations

E1.4 show how changing one or more dimensions of a two-dimensional shape and a three-dimensional object affects perimeter/circumference, area, surface area, and volume, using technology when appropriate

Cross Curricular Connections

Science: Interpret data/information to identify patterns and relationships

Geography: Patterns and trends

Learning Goals and Success Criteria:

LG1: We are learning to identify what changes and what stays the same in a growing pattern.

- SC1: I can create the next figure in a pattern.
- SC2: I can describe, in words, how the pattern is growing.
- SC3: I can see and describe a pattern in more than one way.

LG2: We are learning to determine the general case shown in a geometric growing pattern by determining an equation that connects the number of shapes in the figure to the figure number.

- SC1: I can interpret the pattern visually and draw the general case.
- SC2: I can represent the general case for each different way of seeing the pattern with an algebraic expression that represents that way of seeing the pattern.
- SC3: I can connect the different general cases and prove that they are equivalent by simplifying their equations.

LG3: We are learning to simplify algebraic expressions to show that equations are equivalent.

- SC1: I can combine like terms.
- SC2: I can apply the distributive property to an algebraic expression.
- SC3: I can verify my equation by solving it for different figure numbers.

LG4: We are learning to determine the general case in a growing pattern that is connecting the length of the perimeter of a figure to the figure number.

SC1: I can determine the perimeter of each figure in a growing pattern.

SC2: I can represent the general case for each different way of seeing the perimeter of a geometric pattern with an algebraic expression that represents that way of seeing the pattern.

CONSIDERATIONS THROUGHOUT THE LESSON

Differentiated Instruction and Universal Design for Learning

Use Visibly Random Groupings to create small groups.
Use flexible small group instruction for students who need support.
Consider previewing vocabulary: polygon, trapezoid, hexagon, expression, simplifying, equation, solving, like terms, distributive property, perimeter, generalise/general case.
Construct the sequence of patterns to address students who are at different places in their learning.
Provide access to manipulatives (e.g., colour tiles, linking cubes) so students can see and touch the squares.
Consider Extension Opportunities for groups who finish early.

Assessment

Throughout the lesson, the teacher will be listening for students correctly and effectively using mathematical language to describe their mathematical thinking.
Listen for understanding of the structure of visual growing patterns.
Are students able to represent the pattern using numerical expressions connecting the number of shapes in the figure to the figure number (rather than to the number of shapes in the previous figure)?
Are students able to generalise from the numerical expressions to the corresponding algebraic expression for the general case? Is anyone struggling to connect the abstract representation of the figure number (n) to the pattern?
Are students seeing the connections between the patterns and their corresponding algebraic expressions and equations?

Students will self evaluate using the Success Criteria.

Exit ticket (see Consolidation section)

RESOURCES AND LEARNING ENVIRONMENT

Educator Resources Needed

For more examples of patterns that can be used in this lesson, teachers can visit the website Visualpatterns.org

Student Materials Needed

Whiteboards/chalkboards/windows as vertical non-permanent surfaces, if possible.
Dry-erase markers
Manipulatives such as colour tiles (preferably squares) or linking cubes
Coloured pencils/crayons/markers

Learning Environment Considerations

The Minds-On section allows students to refresh their skills on several topics: combining like terms, distributive property and determining the perimeter of a polygon. The vocabulary should be established during this time. Students are working in flexible groups and sharing as a whole class.

The Action will begin with the teacher leading a whole group investigation of a given pattern. Then students will work in small groups (visible random groups of 2-3 students). Each group will need space to work with a vertical non-permanent surface. They should be able to communicate with other groups and compare ideas as they are working.

The Consolidation will be completed as a whole class. The Exit Ticket will be completed individually.

LESSON CONTENT

Minds-On (10-15 Minutes)

Students should work in flexible groups. Once each part has been completed, strategies can be shared with the whole class.

Part 1:
Determine a simplified expression for the perimeter of each shape. Be sure to show your work.

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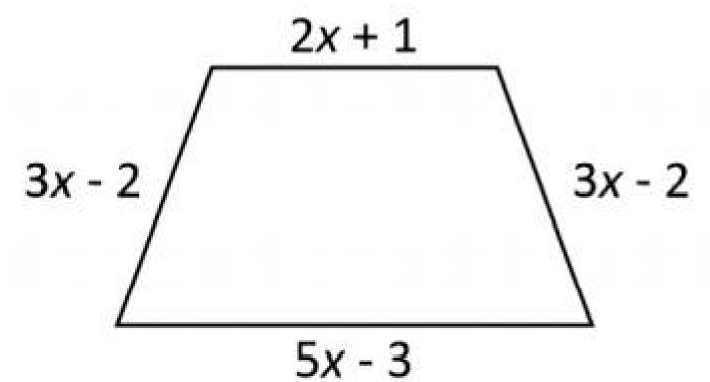

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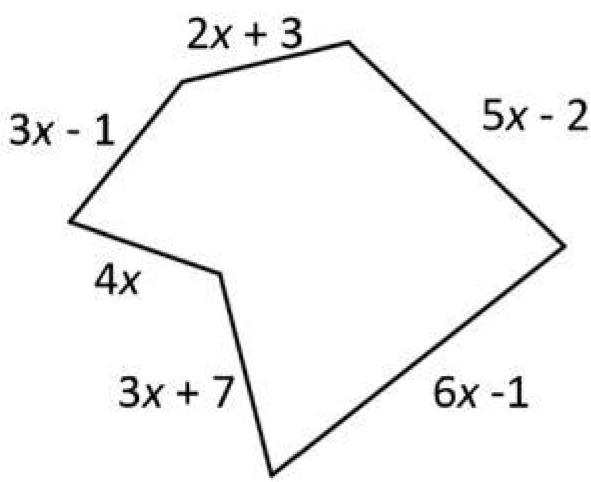


Part 2:

Students will do the Matching Game to review combining like terms and the distributive property.

Teachers can use the results of the Matching Game to provide more support throughout the lesson to students who need it. Students should understand that there can be many algebraic expressions that are equivalent and they should be given the opportunity to practice showing equivalence.

Action (75-90 Minutes)

Pattern 1

The Action begins with a whole group activity: Work through this first pattern with students. Give each student a worksheet with the first 3 figures. Project the pattern or otherwise share it with the whole class.

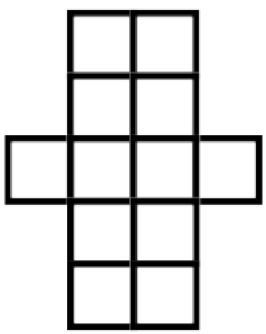


Figure 1

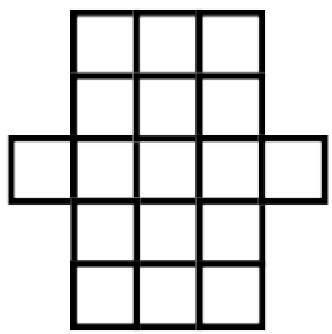


Figure 2

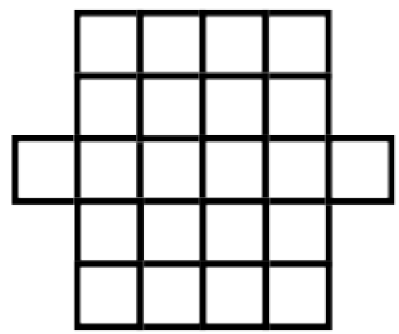


Figure 3

Ask the following series of questions and have students share their responses.

What is changing from one figure to the next? Imagine that you are describing it to someone who cannot see it:

- What stays the same? Collect student responses. (Anticipating student responses: the general shape, the number of rows, the longer row is in the middle, the symmetry, there are always two tiles sticking out in the middle row, …)
- What is not the same? What changes from figure to figure? Again, collect student responses. (Anticipating student responses: the perimeter, the width of the middle part, the area, the number of columns, the number of tiles/squares needed/used, the figure number, …)

Share with students that for this particular pattern, we are looking at how we can see the pattern in the total number of tiles/blocks in each figure by deciding on what to focus our attention on and asking: “What stays the same? What changes? What ways do you see a way to describe the pattern by referring to things that don’t change and to other things that do change?” Allow time for students to reflect on this. If students are having difficulty, the group might cross off items from the previous two lists that do not apply to the pattern in the number of blocks in each figure.

What would the next figure, Figure 4, look like? Give students a minute or so to represent the next figure on their sheet by drawing or using blocks/tiles, then have students describe to you what to draw. Listen for clear communication using mathematical language.

Before students move to small groups, provide the following instructions for their next task:

Think about what Figure 17 would look like. Many of you will be thinking about the pattern in different ways. In the way that **you** see the pattern, how might you decide how many tiles this figure will have in total? Does it help you to think about how many tiles there would be in the part(s) that stay the same? How might we decide how many tiles are in the parts that do change?

Show how you and your group members are thinking about this pattern by colouring the part(s) that stay the same in one colour and the part(s) that change in another colour.

Thinking about how **you** see the pattern, how could you write a general expression for the number of tiles/blocks in Figure ‘n’?

Teachers should ensure that students are not simply counting the total number of blocks. Discourage the use of a table of values at this point to ensure the focus remains on the visual pattern so students can see what is changing and what is staying the same from one figure to the next. The goal is to connect the number of blocks to the figure number so that the general case can be written using an equation. Students can test their equation by seeing if it produces the same number of blocks as shown in Figure 3 and in their drawing of Figure 4.

Anticipating students responses:

1st possibility:

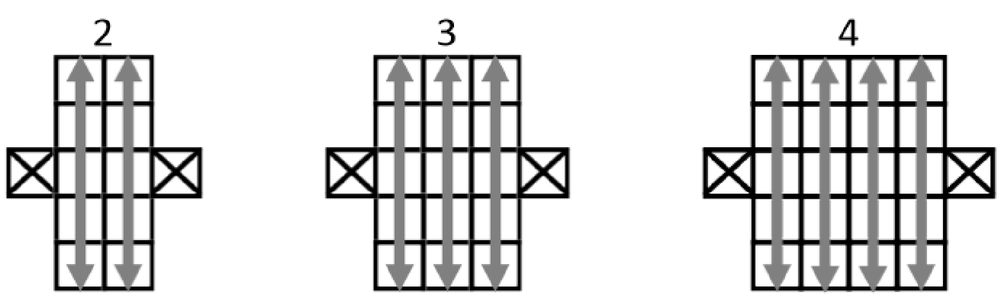


Figure 1 Figure 2 Figure 3

Generalizing, there are 2 sets of 5 vertical blocks at Figure 1, 3 sets at Figure 2 and 4 sets at Figure 3 so each time there is one more set of vertical blocks than the figure number. In addition, there are 2 blocks that are constant.

Corresponding general case:

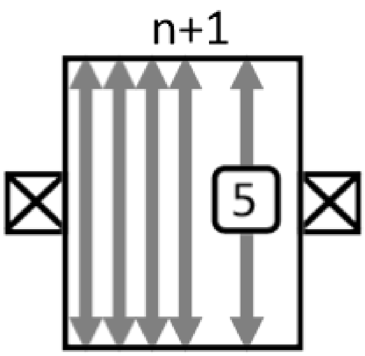


Figure n

$2 + 5(n+1)$

Equation: $y = 2 + 5(n + 1)$

2nd possibility:

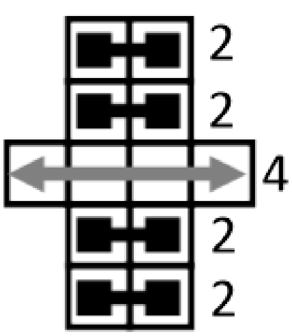


Figure 1

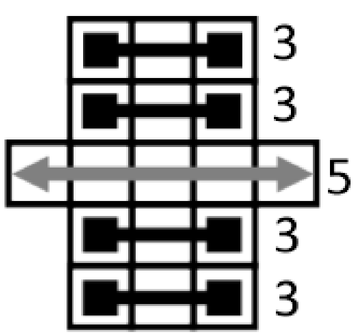


Figure 2

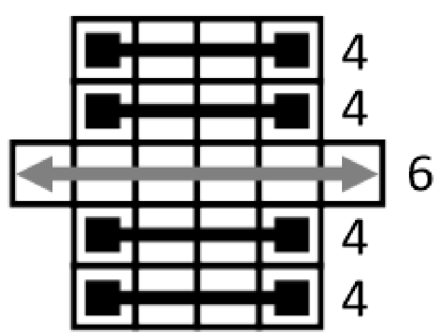


Figure 3

Generalizing, there is a set of 4 blocks across the middle at Figure 1, 5 at Figure 2 and 6 at Figure 3 so in each case there are 3 more blocks than the figure number. There are also 4 sets of 2 blocks at Figure 1, 4 sets of 3 blocks at Figure 2 and 4 sets of 4 blocks at Figure 3 so in each case there are 4 sets of blocks whose length is one more than the figure number. Nothing is constant.

Corresponding general case:

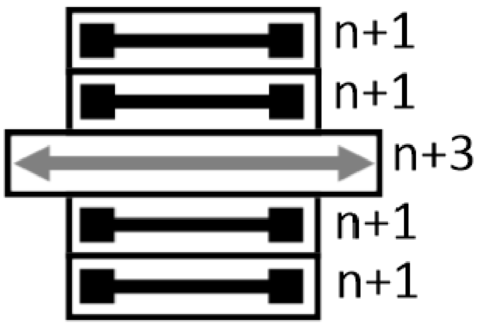


Figure n

n+1+n+1+n+3+n+1+n+1

Equation: y = n + 1 + n + 1 + n + 3 + n + 1 + n + 1

3rd possibility:

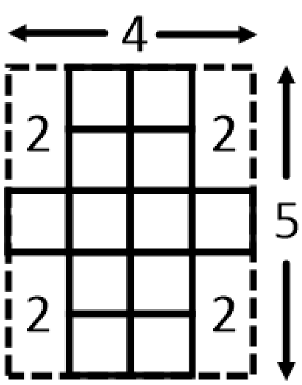


Figure 1

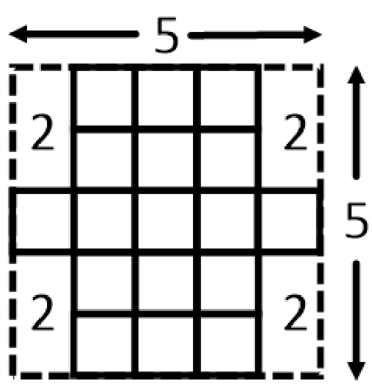


Figure 2

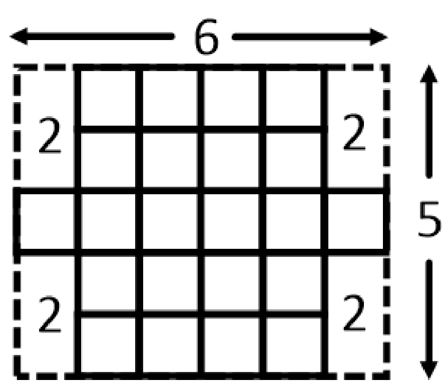


Figure 3

Generalizing, there is a 4 by 5 rectangle at Figure 1, a 5 by 5 rectangle at Figure 2 and a 6 by 5 rectangle at Figure 3 so in each case the width is 3 more than the figure number and the length is 5. Each rectangle has a constant 2 blocks missing from each corner.

Corresponding general case:

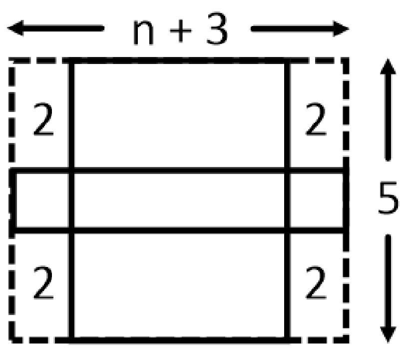


Figure n

$$(5)(n+3) - 4(2)$$

Equation: $y = 5(n + 3) - 4(2)$

Other ways of seeing this pattern exist. Teachers should encourage students to see the pattern in as many ways as possible. It is important to spend time showing the different ways of seeing the pattern and how to find the general case.

As a whole group, elicit from groups the way(s) in which they have "seen" the pattern. If they are able, have them share their equation for the general term. If not, have them share how they thought about their pattern and how they would determine how many blocks would be in Figure 17.

Consolidate this introduction by having students work in their groups to demonstrate that all the equations are equivalent.

Geometry and Measurement Connection for Pattern 1

Begin as a whole class. Introduce the next activity with the following scenario:

Using the same pattern, suppose that each block has a length of 1 unit. How would you determine the perimeter of each figure?

Teacher prompts: When we are using tiles like this, how do we measure the perimeter? What parts of the perimeter are the same for each figure? What parts of the perimeter are changing between figures? How does the changing part of the perimeter relate to the figure number?

Once it is clear that all students understand how to determine the perimeter of the three given figures, have them work in visibly random groups of 3 at vertical, non-permanent surfaces, if possible, to determine a pattern in the change in the perimeter. Coloured tiles should still be made available to all groups. The teacher should circulate and observe, using the prompts below as needed. Students should start by determining the perimeter of Figure 4, which they have from the last part of the Action. Then they should look for a pattern in the way the perimeter is changing from figure to figure.

Teacher prompts: When you are looking at the perimeter of each figure, what stays the same in each figure? What is changing? How is it changing?

Students should verbally explain to each other how they see the pattern. Teacher prompts: Use colour to show the parts that stay the same and the parts that are changing. Label any side lengths or number of blocks appropriately

Then students should use their pattern to determine an equation for the relationship between the figure number and the perimeter of that figure. They can then verify that their equation works by checking it against the expected perimeter. Finally, students should investigate if the formula for the general case can be written in a simpler way.

They should then draw the shape for Figure n and determine an equation for the general case for the pattern that relates the perimeter (y) to the figure number (n). Teacher prompt: Can the expression for the general case be written in a simpler way?

More Patterns

Students will now practice these skills by examining new visual patterns, looking for patterns in the number of tiles as well as in the perimeter of each figure.

Repeat all of the steps for each different way of seeing the pattern.

After students have found multiple ways of seeing the pattern, they should prove equivalence by combining like terms and using the distributive property.

The teacher should keep track of which patterns each group has successfully completed and should choose one pattern that all students have completed for the consolidation. Ensuring that each different way of seeing this pattern remains on the VNPS or is recorded with a photo will help the consolidation go smoothly.

Sample patterns:

Pattern #1:

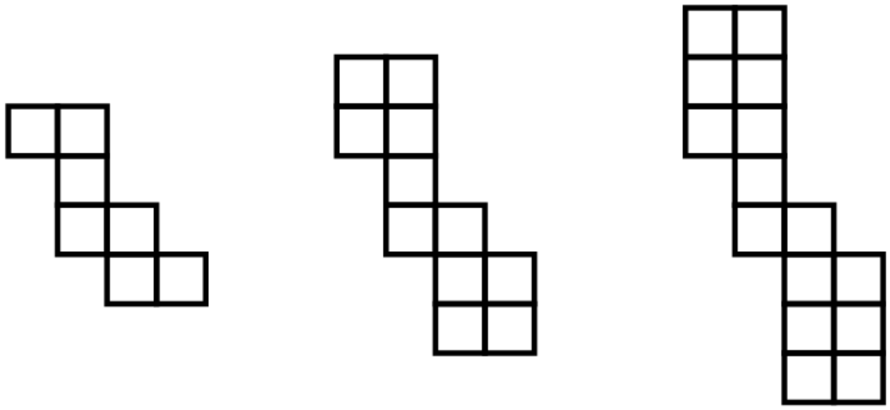


Figure 1 Figure 2 Figure 3

Anticipating possible ways that students will see the pattern for total number of tiles (which they may also use for perimeter):

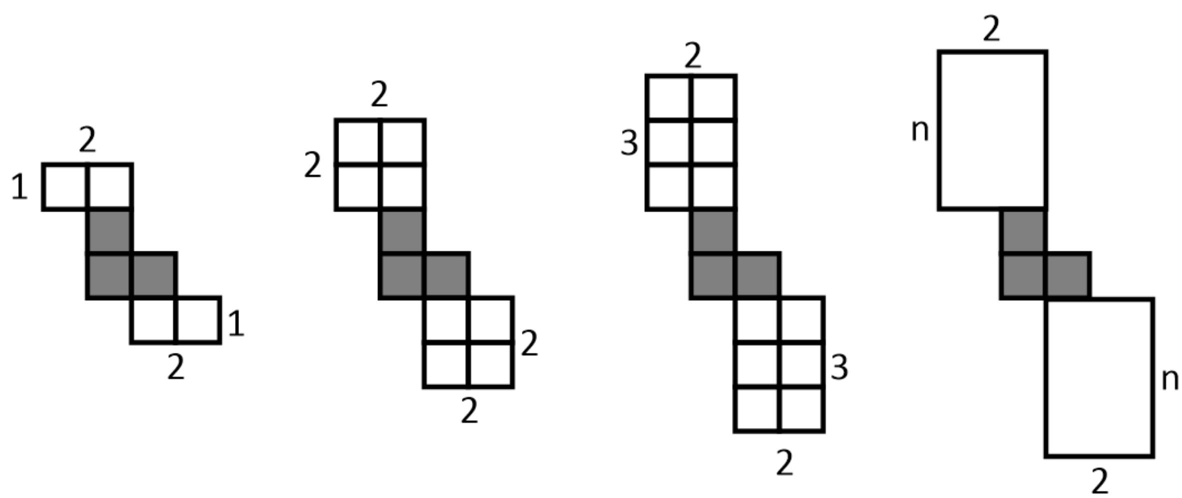


Figure 1 Figure 2 Figure 3 Figure n

2n + 2n + 3

or perhaps 2(2n) + 3 (looking at the symmetry)

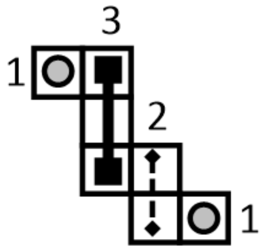


Figure 1

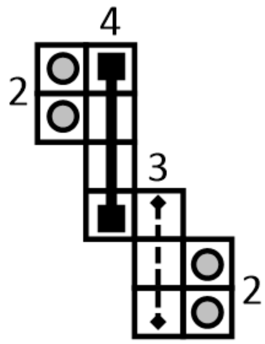


Figure 2

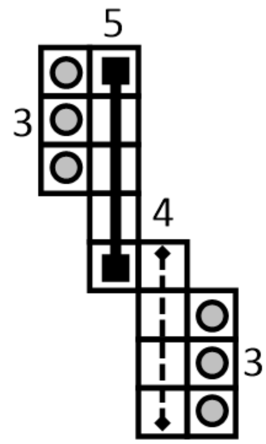


Figure 3

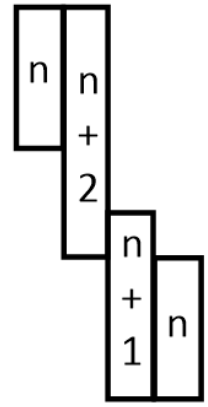


Figure n

n+n+2+n+1+n

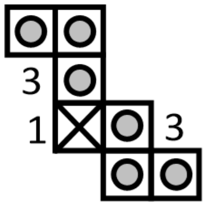


Figure 1

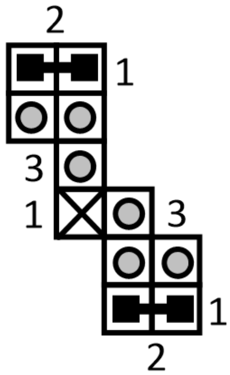


Figure 2

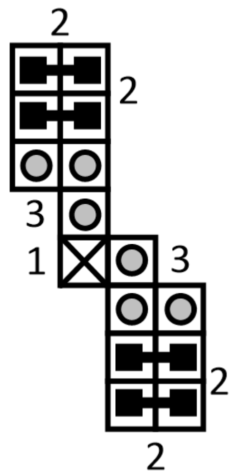


Figure 3

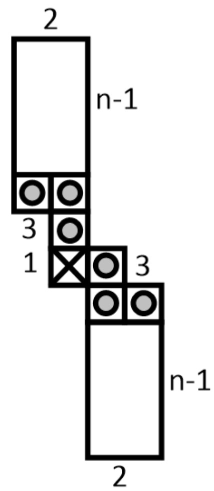


Figure n

2(n-1)+3+1+3+2(n-1)

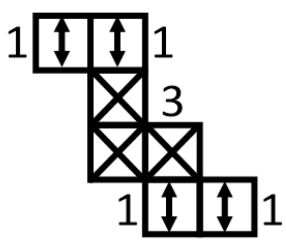


Figure 1

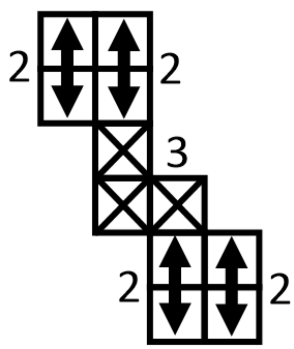


Figure 2

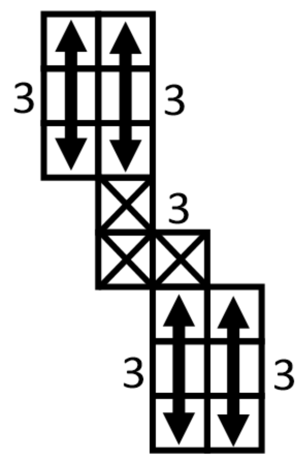


Figure 3

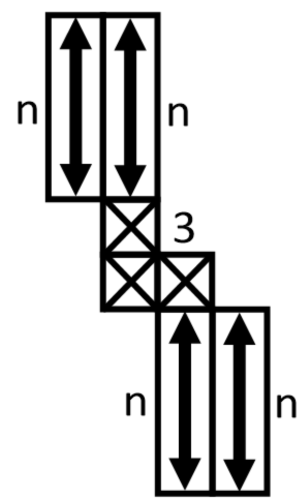


Figure n

4n+3

Pattern #2:

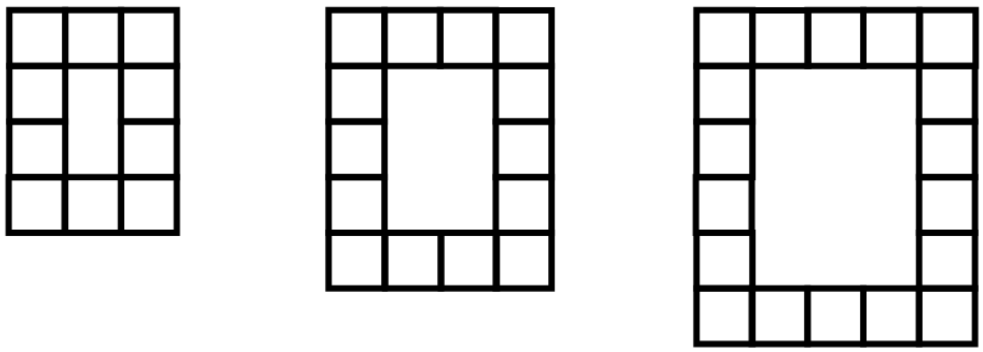


Figure 1 Figure 2 Figure 3

Anticipating possible ways that students will see the pattern for total number of tiles (which they may also use for perimeter):

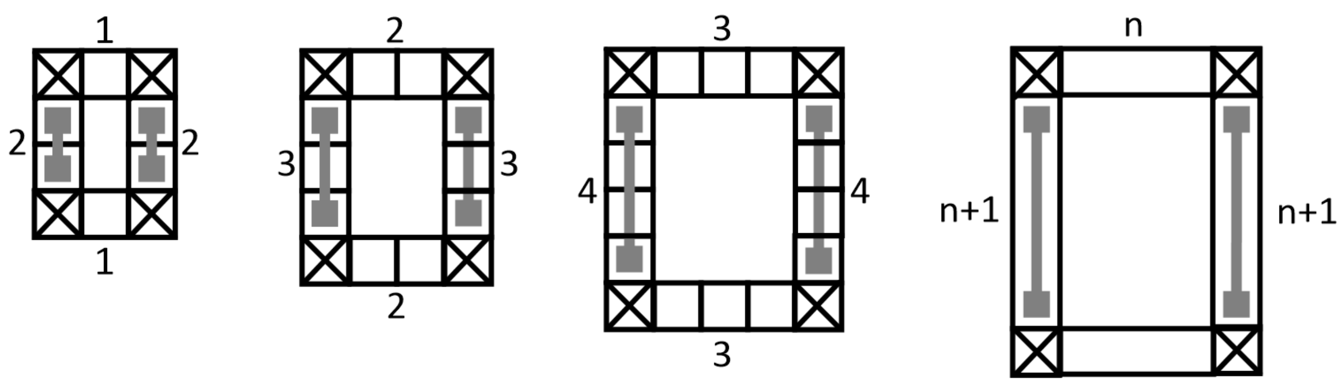


Figure 1 Figure 2 Figure 3 Figure n

4+2(n+1)+2(n)

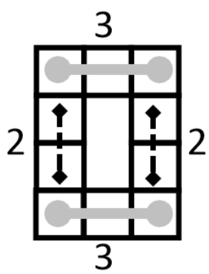


Figure 1

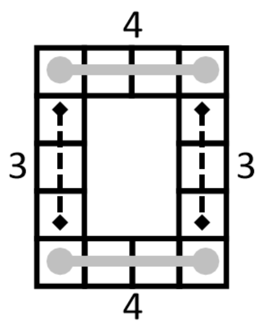


Figure 2

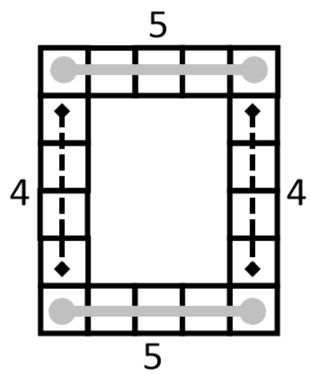


Figure 3

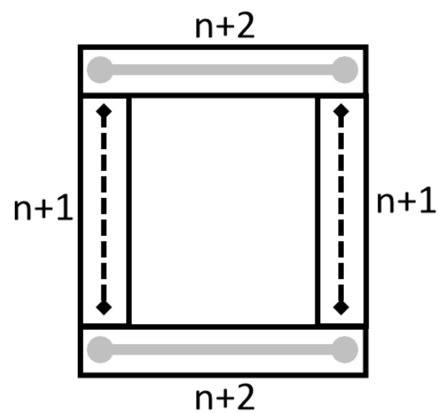


Figure n

2(n+2)+2(n+1)

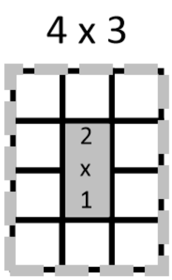


Figure 1

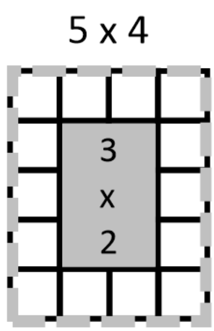


Figure 2

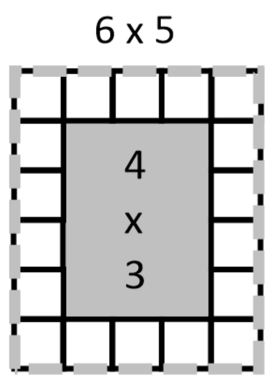


Figure 3

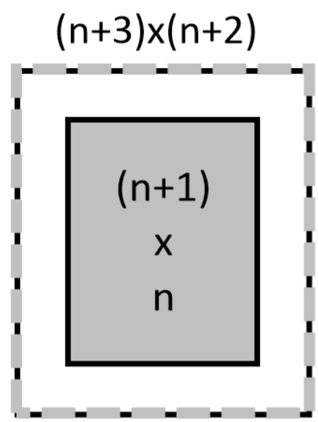


Figure n

[(n+3)x(n+2)]-[(n+1)xn]

Pattern #3:

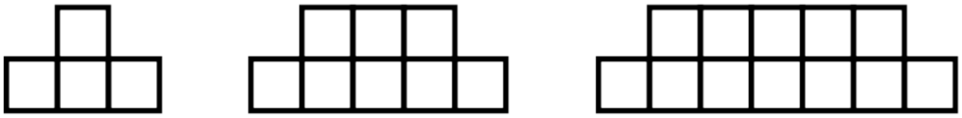


Figure 1 Figure 2 Figure 3

Anticipating possible ways that students will see the pattern for total number of tiles (which they may also use for perimeter):

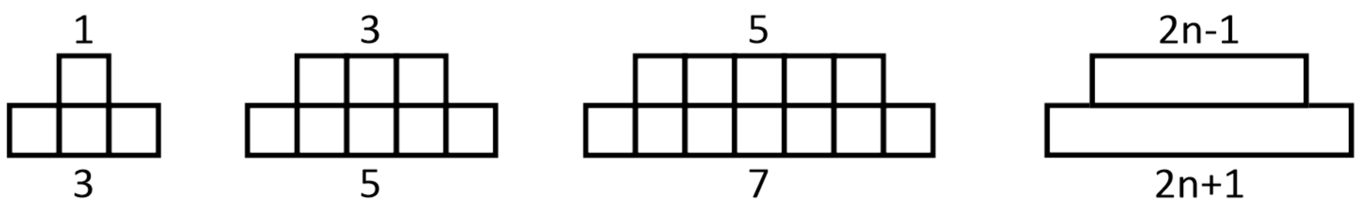


Figure 1 Figure 2 Figure 3 Figure n

$2n-1+2n+1$

Extension Opportunities

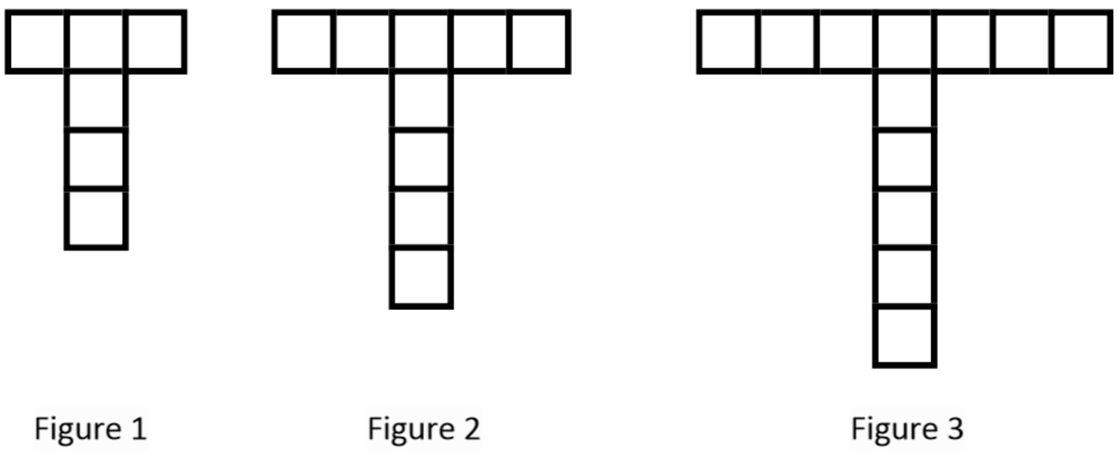
Link to [Visual Patterns - 1-20] for more patterns. Be sure to choose linear patterns.
Change the Figure numbers to 2, 3 and 4. How does that change the general case?
Create your own pattern.
Create a shrinking pattern.
Determine the perimeter of each figure and the formula for the perimeter of any figure if each block has side length that is a fraction between 0 and 1.
Determine the figure number that will produce a certain number of blocks using the equation for the general case.

Consolidation (15 Minutes)

As a whole class, go over a pattern that all students completed. Select one for which students looked at the total number of tiles and one for which they looked at the perimeter. Students can share their reasoning and approach for each different way of seeing the pattern. The teacher can model other ways of seeing the pattern once students' methods have been completed. Explain how to get the general case for each way of seeing the pattern. It is important to have multiple ways of seeing the pattern grow. Write the algebraic expression corresponding to each general case. Students should return to their groups and, with the help of the distributive property, simplify all the expressions that the class has found. This should prove that they are all equivalent. The collection of algebraic expressions should allow a recap of simplifying algebraic expressions by collecting like terms and by using the distributive property.

Ask the class to brainstorm and share ideas of where they see growing patterns in the world around them (for example: in nature, in finance, in health). Encourage students to share examples from their cultures and experiences. This is an excellent place to introduce mathematical modelling, in which students will be encouraged to identify a situation, select a question about that situation that is of interest to them, and then explore the patterns in the relationships they find. Some sample modelling tasks that could be used as an introduction to mathematical modelling are: the pattern found over time between year and age at which marriage occurs, the relationship between shoe size and foot length, the relationship between the outside temperature and the speed at which a cricket chirps.

Exit Ticket:
1. Determine at least two ways of seeing the pattern in the total number of tiles in each figure for the following visual pattern. Determine the general case for each. Then, prove that the algebraic expressions are equivalent.



2. Using the same visual pattern, given that each tile has a length of 1 unit, determine at least two ways of seeing the pattern in the perimeter. Then, determine the general case (perimeter of Figure n) for both ways of seeing the pattern, and prove that both expressions are equivalent.

Appendices

Pattern 1

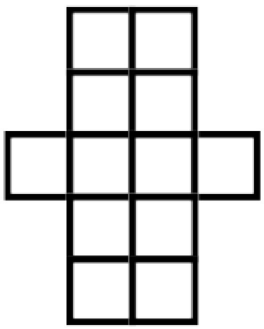


Figure 1

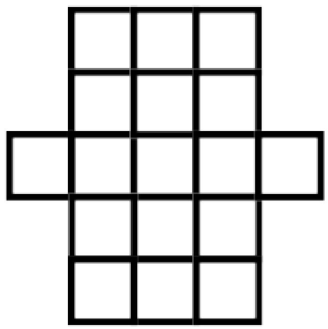


Figure 2

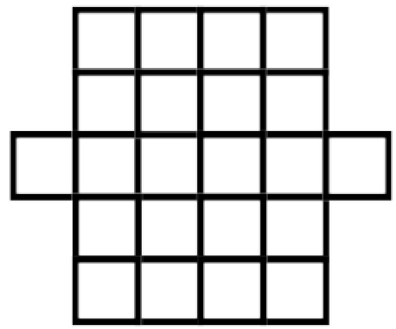


Figure 3

Figure 4

Appendix 2

This table of values shows the pattern for determining perimeter for Pattern 1 - it is not intended to be shared with the students. It is intended for reference only.

Figure #	Perimeter (units)
1	18
2	20
3	22
n	$14 + 2(n+1) = 16 + 2n$