LESSON DETAILS

Just Off by a Fraction!

Lesson Summary

In this lesson, students will be working with fractions, decimals, percentages, rates, and proportions to solve problems involving measurement, geometry, and linear trajectory.

Grade 9

Big Ideas

Small errors in inputs can create large and unintended consequences in the outputs. While estimation is often useful, in some situations, precision is of critical importance.

Learning Expectations

AA1. develop and explore a variety of social-emotional learning skills in a context that supports and reflects their learning in connection with the expectations across all other strands

- Developing critical and creative mathematical thinking
- Developing a healthy mathematical identity through building self-awareness

A1. apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- Problem solving
- Connecting

A2. make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers

B3. apply an understanding of rational numbers, ratios, rates, percentages, and proportions, in various mathematical contexts, and to solve problems

> **B3.3** apply an understanding of integers to explain the effects that positive and negative signs have on the values of ratios, rates, fractions, and decimals, in various contexts

B3.4 solve problems involving operations with positive and negative fractions and mixed numbers, including problems involving formulas, measurements, and linear relations, using technology when appropriate
B3.5 pose and solve problems involving rates, percentages, and proportions in various contexts, including contexts connected to real-life applications of data, measurement, geometry, linear relations, and financial literacy

C1. demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations
C1.5 create and solve equations for various contexts, and verify their solutions

E1. demonstrate an understanding of the development and use of geometric and measurement relationships, and apply these relationships to solve problems, including problems involving real-life situations
E1.3 solve problems involving different units within a measurement system and between measurement systems, including those from various cultures or communities, using various representations and technology, when appropriate
E1.5 solve problems involving the side–length relationship for right triangles in real-life situations, including problems that involve composite shapes

Cross Curricular Connections

TIJ1O
A2.3 use metric and imperial units of measurement (e.g., metric: degrees Celsius, joules, micrometres [microns], millimetres, kilohms, L/100 km, tonnes; imperial: degrees Fahrenheit, BTUs, knots, mils, inches, feet, miles per gallon, pounds per square inch, tons) and the abbreviations or symbols associated with them correctly and as appropriate to the task

SNC1P
A1.12 use appropriate numeric, symbolic, and graphic modes of representation, and appropriate units of measurement (e.g., SI and imperial units)
A1.13 express the results of any calculations involving data accurately and precisely

Learning Goals and Success Criteria:

Learning Goals should be revised and modified according to the teacher's vision and intention for the lesson, as well as in consideration of where in the course this lesson is sequenced. The Success Criteria should be revised and modified in collaboration with the students.

LG1: We are learning to perform calculations with integers, fractions, percentages, rates, and proportions in context.

- SC1: I can use proportions to convert units.
- SC2: I can identify the rate of change and use it to solve a problem.
- SC3: I can accurately solve computations involving positive and negative numbers, whether they be integers, fractions, decimals, or percentages.
- SC4: I can solve an equation for an unknown quantity.

LG2: We are learning to interpret situations in mathematical terms.

- SC1: I can correctly select a formula or equation to help me solve a problem presented within a real life context.
- SC2: I can identify the given quantities that are relevant to my problem solving strategy.
- SC3: I can identify circumstances that require a unit conversion.

LG3: We are learning that very small inaccuracies when measuring or making computational decisions can create real consequences in real world situations.

- SC1: I can explain why small inaccuracies have created problems in the context of real world situations.

LG4: * We are learning to collaborate with peers in order to overcome challenges.

- SC1: I can demonstrate an openness to the ideas of others.
- SC2: I can participate actively in the discussions in the group.
- SC3: I can reach a consensus with my group in order to solve a challenge.

* Learning Goal 4 and the associated Success Criteria should be used to collect evidence related to work habits and skills, not socio-emotional learning as outlined in MTH1W. These evaluation criteria must therefore not affect the student's grade in the course.

CONSIDERATIONS THROUGHOUT THE LESSON

Differentiated Instruction and Universal Design for Learning

- Use vertical non-permanent surfaces for students to work at, encouraging cross-pollination of ideas and strategies
- The lesson provides for teacher-led group discussion, independent working time (with a partner), and whole group sharing.
- Allow for a significant period of quiet time after asking a question to the whole class, providing time for each student to consider the question and formulate their ideas before hearing the interpretations and ideas of other students.
- Students will be working in visibly random groups during the Action.

Assessment

In the Action, after each presentation, students are invited to share questions, comments, or concerns they have with the presenting team. This will provide feedback to the presenting students as to the completeness of their solutions and clarity of their communication.

After all the presentations have been given, all students are provided with an opportunity to rejoin their partner to complete a final edit on their written solution before they have to hand it in to the teacher for feedback. This supports students' development of self assessment and metacognitive skills.

The teacher will assess the students' achievement of the learning goals using the evidence observed while the teams are working, during the presentations, and the work that has been handed in. A Success Criteria Checklist has been included in the appendices of this lesson. (Note: If the Success Criteria have been changed in collaboration with the students, these should be changed here as well.)

Students will receive feedback on their written solutions.

Self Assessment against Success Criteria (Note: If the Success Criteria have been changed in collaboration with the students, these should be changed here as well.)

RESOURCES AND LEARNING ENVIRONMENT

Resources required for teaching staff

A piece of 2 by 4 lumber
Copies of Team Task Cards (cut into individual cards) Ten cards are provided, but more may be necessary
Opaque container
Mathematical Processes Posters, Tent Cards, Bookmarks, or Role of Students sheets
Success Criteria Checklist

Materials needed by students

Team Task Card
Mathematical Processes Posters, Tent Cards, Bookmarks, or Role of Students sheets
Self Evaluation against Success Criteria

Learning Environment Considerations

During this lesson, students will be asked to focus on developing critical and creative mathematical thinking, and developing a healthy mathematical identity through building self-awareness.
In particular, the following will all be valuable during the research component of the Action (from the Curriculum Context):

Developing critical and creative mathematical thinking will include students:

- Making connections
- Making decisions
- Evaluating choices, reflecting on and assessing strategies
- Communicating effectively
- Managing time
- Setting goals and making plans
- Applying organizational skills
- Applying strategies such as:
  - determining what is known and what needs to be found
  - using various webs, charts, diagrams, and representations to help identify connections and interrelationships

Developing a healthy mathematical identity through building self-awareness will include students:

- Having a sense of mattering and of purpose
- Identifying personal strengths
- Having a sense of belonging and community
- Applying strategies such as:
  - building their identity as a math learner as they learn independently as a result of their efforts and challenges
  - monitoring progress in skill development
  - reflecting on strengths and accomplishments and sharing these with peers or caring adults

During the Action, students may need to look up some terms. Before proceeding to the Presentations section of the Action (Part 3), inquire what terms the teams had to look up and review this new vocabulary with the class. The use of a word wall with visuals would provide ongoing support that benefits all learners.

Consider “de-fronting” the classroom, allowing students to sit in groups rather than in rows facing the front of the room. This helps send the message to students that the space is safe and it is okay to not be perfect.

LESSON CONTENT

Minds-On (15 - 20 minutes)

Whole Class - Number Talk

To set up this Math Talk: Ideally, have the students clustered around the projection of the problem, but they can also be at their desks.

Always present these computation problems horizontally (as presented) so as to encourage flexible thinking and a variety of strategies and to discourage over-reliance on standard algorithms.

Present the problems, one at a time, to the whole class.

Give students 1 minute for each question to **silently and mentally** figure out an answer and a description of their strategy for coming up with an answer. Students are asked to quietly signal to the teacher (eg. with a thumb up against their chest) when they are ready to share, but they do not call out. While they are waiting for their classmates to also indicate they are ready, students are encouraged to think of different ways to solve the problem. Encourage students to identify any connections they may be making.

Ask a few students to share their answers, one by one. Record their answers on a board or chart paper without signalling to the students any approval or disapproval.

The next step is to ask students to describe **how** they got their answers (their strategy). If Math Talks are new to your students, it may be valuable to begin by having students share with a partner, before sharing with the larger group to allow them to practice describing their thinking orally. The teacher **records the student's thinking** and attaches their name to the solution. Some clarifying questions may be needed so that the teacher accurately captures students' strategies. Before moving to another student, verify that you have accurately captured their strategy.
As the students are sharing their thinking with the whole group, the teacher may ask probing questions or offer prompts to help them express themselves more clearly, using proper mathematical terminology. Multiple ways of solving problems and the connections between them are emphasized.

Using the "5 Practices for Orchestrating Productive Mathematics Discussions" (Smith & Stein), the teacher will have anticipated student responses as part of planning the lesson.

In this way, teachers will have already considered how to best sequence the students' strategies so that their thinking is moved towards the learning goals for the lesson. Note that there is **not** one correct way to sequence students' responses.

The idea is to get students to reflect, plan, and communicate their OWN thinking, and to listen to the variety of ideas that come from other students. This is NOT about right and wrong answers, nor about best procedures. It is about honouring the diversity in student thinking and student voice by giving them a part of the lesson in which their strategies are at the forefront. The correct answers will emerge.

If any students offer "two negatives make a positive", follow up by adding one more computation problem to the Math Talk.

$$-\frac{1}{4} + \quad -\frac{3}{4}$$

Action - Part 1 (10 to 15 minutes)

Whole Class:
Present students with this information:

> In its rough-cut condition, a piece of "2 by 4" lumber measures 2 inches thick and 4 inches wide. When wood is milled from a rough to a smooth surface, it loses about ¼-inch from each of its four sides.

Ask the students, "What, then, are the dimensions of a piece of 2 by 4 that you buy from the building supply store?"
Students may do repeated subtraction, or they may collect the ¼-inch from each side as ½ inch per side, or they may have another way of reasoning.
Discuss why 4 - ¼ - ¼ is equivalent to 4 - 2(¼) or 4 - ½. Do not accept "because the answers are the same" as a response. Probe - why are the answers the same?

Students will consider how this misleading label of 2 by 4 could affect the work of a contractor who was unaware of this discrepancy:

> Ask students to identify an elbow partner. Each will assume the title of Partner 1 or Partner 2. Each partner will do a calculation and describe to their partner what percentage less than the expected the contractor will have purchased. Partner 1 will work with the "2" part of the "2 by 4", while Partner 2 will work on the "4" part.

There will likely be different ways of thinking about how to do these two calculations. Elicit several different ways of thinking about this question during a brief discussion. Include in the discussion why the reductions expressed as two percentages are different even though the amount of the two reductions is the same.

After the discussion, pose the next problem: If the contractor needs 100 feet of width and so buys 100/4 or 25 pieces of 2 by 4, what will their shortfall be? Students will be given two methods of thinking about this question:

- Method 1: 25 x 3 ½ subtracted from 100, or
- Method 2: 25 x 4 - 25 x ½

Partner 1 will describe to Partner 2 the thought process that leads to the first of these two methods of answering this question. Partner 2 then describes the thought process of the second method.

Ask one student from each pair to share what they told their elbow partner or to confirm that what they have already heard matches their own thinking. Finally, have students describe why these two ways of thinking, though giving a different perspective, are really equivalent.

(If the distributive property has been previously addressed in class, this would be a nice opportunity to recall how this property allows you to demonstrate the equivalence mathematically.)

Action - Part 2 (40 minutes)

Introduce the second part of the Action by asking if anyone is aware of any stories of disasters happening due to incorrect measurements or miscalculations. If any are offered, record them in a central location that is visible to everyone.

Present the following story:

> "The Hubble telescope is famous for its beautiful space images, and is considered a great success for Nasa. However, it got off to a very rocky start. The first images sent back by the telescope were fuzzy because the telescope's main mirror was too flat. It wasn't out by much - only 2.2 microns, or about 1/50th the thickness of a human hair - but this was enough to put the project in jeopardy. One theory is that a speck of paint on a device used to test the mirror resulted in distorted measurements. Luckily, scientists managed to fix the problem in 1993, using an instrument called the Corrective Optics Space Telescope Axial Replacement (Costar). This cancelled out the error in the main mirror, by matching it in reverse."
> (Source: BBC)

Have an open discussion with the class about how a measurement being off by a really small amount, a fraction of a unit, can have serious consequences.

Cut out the task cards. Place them in a can or jar so that students cannot see what is written on them. Have each student select one card. This will determine their partners. The problems on the cards are repeated. As the students are working on their solutions, look for opportunities to share different perspectives and solution strategies during Part 3 of the Action.
Some tasks may require that students look up terms, but all the required information is given for consistency of answers. The teacher may want to consider extending this activity by removing certain pieces of information (for example, weight of oak or average speed of a cruiser to provide for a more realistic problem-solving experience, and a wider variety of calculations and answers.)
Expect that some of the tasks will take less time to complete by some teams. If enough time remains, ask them to work through another problem or to solve their original problem in a new way.

Let the students know that they will be sharing their solutions orally to the class and that they will have no more than 5 minutes for their presentation. Make sure that students understand that they are sharing their thinking rather than their "steps". Have the mathematical processes posters, tent cards, bookmarks, or Role of Students sheets accessible to the students while they are working to remind them to thoughtfully consider what processes they are using. While they are sharing, listen for correct use of mathematical vocabulary. Highlight the mathematical processes they are using.
After each presentation, allow up to 2 minutes for other students to share with the presenting team any questions, comments, or concerns they have.

Also, let the students know that they will be submitting their written solutions to the teacher for feedback.

Action - Part 3 (20 - 40 minutes, depending on the number of students in the class)

In this final section of the Action, students will share their problems and their strategies for solving them. When different teams have chosen different problem solving strategies or computation methods, highlight the mathematics that connects them.
After all the presentations have been given, provide students another opportunity for students to work with their partner to complete a final edit on their written solution before they have to hand it in to the teacher for feedback.

Students will self assess after all the solutions have been presented.

The teacher will assess the students' achievement of the learning goals using the evidence observed while the teams were working, during the presentations, and the work that has been handed in.

Extension opportunity

Students could create code or use spreadsheets to solve the problems. Since proficiency with computation is one of the goals of the lesson, coding and/or spreadsheets would be used to verify their answers.

Students may be especially interested in exploring more actual instances of small errors causing unintended and possibly catastrophic consequences. Imprecise measurements, unit confusion, and incorrect rounding have accounted for many disasters throughout history. These are easy to search for. For some students, this could lead to a B1.1, C1.1, D1.1, E1.1, or F1.1 exploration.

The teacher may use this lesson as an entry to a longer mathematical modelling investigation in which precision and/or optimization is a key concept.

Consolidation (15-25 minutes)

Show students a short clip from the Formula 1 episode of Richard Hammond's Engineering Connections. Cue it to time 2:14 and play it to time 10:04.
Summary: "Attaining huge speeds requires a precision-built engine, which maximises its power thanks to a revolutionary cannon, (which is like an open-ended engine cylinder). Richard fires his own home-made cannon to show how minimising what gunners called 'windage', the gap between the cannon ball (or piston) and the barrel (or cylinder) increases the power of the shot (or engine)."
(Source of summary: BBC)

The episode tells us that Formula 1 racing technology uses some of the same principles as military artillery. It is important to have the right fit between the projectile and the cylinder from which it is being fired. Too much space and you lose power through the gap. Too little and things get stuck. Enter "the windage gap". In this clip, we see that when they get the gap right, they get an extra 12 meters of distance from a cannon, an increase in range of 25%. Precision. A tiny difference makes a really big difference.

The same principle of "just the right fit" applies in many other circumstances. For example, a popular cosmetics company sells shampoo bars -- perfect for travelling because they are not liquid. To accompany the shampoo bars, they also sell metal containers in which to store the bars. If they make them too small, the bars will break trying to force them into the container. If they make them too big, they will be spending money on raw materials, storage, and shipping costs unnecessarily. Precision is very important to this business.

After showing the video clip and the short passage, lead a discussion about what they noticed and wondered about what they saw and heard, then transition to the reflection.

Reflection question:
“Someone has asked you how they can determine under what circumstances might it be perfectly fine to use an estimate and when it would be really important to be very precise. What advice would you give them?” Let the students know that it may be helpful to them to select a fictitious situation within which to frame their advice.

Appendices

Appendix 1: Minds-On Number Talk

1. $\frac{3}{4} \times \ 20$
2. $-\ \frac{1}{4} \times \ 20$
3. $-\ \frac{1}{4} \times \quad -\ \frac{3}{4}$

Anticipated student responses

1. Response 1: "I changed 20 to 20 over 1, then I multiplied the two numerators, 3 x 20 and got 60, then multiplied the two denominators, 4 x 1, and got 4. So then I had 60 over 4. Then I thought that 60 over 4 needs to be reduced by dividing the numerator and denominator by 4, so I got 15 over 1 which is 15."

   Response 2: "I thought that I could associate the 4 with the 20, so I had 20 divided by 4, which is 5, so then I had to multiply the 5 by the 3 and I got 15."

   Response 3: "I was thinking that if I had 20 squares and set them up in an array that has 4 rows, each row would have 5. Then I colour in 3 of the rows and I have 5 + 5 + 5 which is 15."

   Response 4: "I thought of 20 as 4 times 5 and then I cancelled the 4 with the 4 in the 20 and got 3 times 5 which is 15."

2. Response 1: "I thought that I could do the same thing as (Response 1) did and I got 1 times 20 in the numerator and 4 times 1 in the denominator so 20 over 4, which is 5. But there is one negative sign so it's negative 5."

   Response 2: "I associated the negative with the 1, and negative 1 times 20 is negative 20, so I got negative 20 divided by 4 which is negative 5."

   Response 3: "I just thought that this is the same as the last question, only it's a third as large, so instead of 15 it's 5. But then the negative reflects it over 0 to the negative numbers, so the answer is negative 5."

3. Response 1: " I multiplied the two numerators, 1 times 3 and got 3, then multiplied the two denominators, 4 times 4, and got 16. So then I had 3 over 16 and that doesn't reduce. Then I thought that two negatives make a positive so I left it as 3 over 16."

   Response 2: "I multiplied the two numerators, negative 1 times negative 3 and got 3, then multiplied the two denominators, negative 4 times negative 4, and got 16. So then I had 3 over 16 and that doesn't reduce."

   Response 3: "I put the negatives in the numerators and multiplied the two numerators, negative 1 times negative 3 and got negative 3. Then I multiplied the two denominators, 4 times 4, and got 16. That's 3 over 16. Three doesn;t go into 16 evenly, so I left it as 3 over 16."

Appendix 2: Task cards

TEAM 1 The radius of a cylindrical can was designed to be 16 cm wide, but it has come off the assembly line exactly 1/10 cm less wide than designed. The height is 10 ¼ cm, exactly as designed. The labels are already printed and they show that the volume of the sauce that is going into the can is 2 litres. The food processing plant needs to know if the cans can be used or if they will need to be discarded, causing major delays in the delivery of their product.	TEAM 2 The radius of a cylindrical can was designed to be 16 cm wide, but it has come off the assembly line exactly 1/10 cm less wide than designed. The height is 10 ¼ cm, exactly as designed. The labels are already printed and they show that the volume of the sauce that is going into the can is 2 litres. The food processing plant needs to know if the cans can be used or if they will need to be discarded, causing major delays in the delivery of their product.
TEAM 3 A moving company has a maximum weight of 120 pounds for items of furniture and boxed items before they will charge a substantial extra fee. A solid oak circular table top has a radius of 1 ⅛ feet and a depth of ¾ inches. It is secured to a cylindrical oak base that has a diameter of 11 ½ inches and is 3 ⅓ feet high. Remember that there are 12 inches in 1 foot. Oak weighs about 45.5 pounds per cubic foot. The family moving this table estimated that the table weighs about 115 pounds and so they have not budgeted for this extra fee. Will they have a budget shortfall?	TEAM 4 A moving company has a maximum weight of 120 pounds for items of furniture and boxed items before they will charge a substantial extra fee. A solid oak circular table top has a radius of 1 ⅛ feet and a depth of ¾ inches. It is secured to a cylindrical oak base that has a diameter of 11 ½ inches and is 3 ⅓ feet high. Remember that there are 12 inches in 1 foot. Oak weighs about 45.5 pounds per cubic foot. The family moving this table estimated that the table weighs about 115 pounds and so they have not budgeted for this extra fee. Will they have a budget shortfall?
TEAM 5 The crew of a cruiser set the course for the pier at which it was supposed to dock. They only have 21 hours of fuel remaining. They used horizontal and vertical components and arrived at their path, which is given by the line y = 540 - 25.75x, where 540 represents their remaining distance and 25.75 represents their speed in km/h. However, they did not take into account the effect of	TEAM 6 The crew of a cruiser set the course for the pier at which it was supposed to dock. They only have 21 hours of fuel remaining. They used horizontal and vertical components and arrived at their path, which is given by the line y = 540 - 25.75x, where 540 represents their remaining distance and 25.75 represents their speed in km/h. However, they did not take into account the effect of

headwinds on their speed which slows their speed by 2.5%. Will they still make it to shore with their remaining fuel? Did the headwind make a difference to the outcome?	headwinds on their speed which slows their speed by 2.5%. Will they still make it to shore with their remaining fuel? Did the headwind make a difference to the outcome?
TEAM 7 An artist has planned a sculpture made of metal rods. Her design features a series of interwoven right triangles positioned at various angles to each other that people can walk through and have an interactive experience with the piece of art. She has ordered three different sized rods, 10 feet, 24 feet, and 26 feet. When they arrive, she finds that they are not cut to these exact lengths. Each rod is ⅛ inches shorter than expected. Will she be able to connect them at right angles? If not, what could you suggest?	TEAM 8 An artist has planned a sculpture made of metal rods. Her design features a series of interwoven right triangles positioned at various angles to each other that people can walk through and have an interactive experience with the piece of art. She has ordered three different sized rods, 10 feet, 24 feet, and 26 feet. When they arrive, she finds that they are not cut to these exact lengths. Each rod is ⅛ inches shorter than expected. Will she be able to connect them at right angles? If not, what could you suggest?
TEAM 9 In the early 1980s, a new stock index at the Vancouver Stock Exchange tracked a steady and mysterious loss in value. An investigation revealed that they had coded to use the “floor” function, which dropped off any decimals past 3, instead of coding to use the “round” function, which would have rounded to 3 decimal places according to the rounding rules we are familiar with. They operated with this code for 22 months, averaging 3000 trades per day. This created a loss of about 25 points per month. The programming mistake was finally fixed; the index closed around 500 on a Friday and reopened the following Monday. What was the approximate value when they reopened? What was the percent increase in value that would have been attributed to the coding fix?	TEAM 10 In the early 1980s, a new stock index at the Vancouver Stock Exchange tracked a steady and mysterious loss in value. An investigation revealed that they had coded to use the “floor” function, which dropped off any decimals past 3, instead of coding to use the “round” function, which would have rounded to 3 decimal places according to the rounding rules we are familiar with. They operated with this code for 22 months, averaging 3000 trades per day. This created a loss of about 25 points per month. The programming mistake was finally fixed; the index closed around 500 on a Friday and reopened the following Monday. What was the approximate value when they reopened? What was the percent increase in value that would have been attributed to the coding fix?

Appendix 3: Self Assessment against Success Criteria

	Achieved? Y/N	**Evidence of achievement or next steps for improvement**
I can use proportions to convert units.		
I can identify the rate of change and use it to solve a problem.		
I can get correct answers to computations involving positive and negative numbers, whether they be integers, fractions, decimals, or percentages.		
I can solve an equation for an unknown quantity.		
I can correctly select a formula or equation to help me solve a problem presented as a real life situation.		

I can identify the given quantities that are relevant to my problem solving strategy.		
I can identify circumstances that require a unit conversion.		
I can explain why small inaccuracies have created problems in the context of real world situations.		

Appendix 4: Success Criteria Checklist

________________________ **has demonstrated that they are able to:**	**Achieved**	**In Progress**	**Follow Up**
use proportions to convert units			
identify the rate of change and use it to solve a problem			
get correct answers to computations involving positive and negative numbers, whether they be integers, fractions, decimals, or percentages			
solve an equation for an unknown quantity			
correctly select a formula or equation to help me solve a problem presented as a real life situation			
identify the given quantities that are relevant to my problem solving strategy			

identify circumstances that require a unit conversion			
explain why small inaccuracies have created problems in the context of real world situations			