LESSON DETAILS

Where We Draw the Line

Lesson Summary

This lesson allows students to represent linear relations using concrete materials, tables of values, graphs, and equations, and to make connections between the various representations to demonstrate an understanding of rates of change and initial values.

Grade: 9

Big Ideas

Linear relationships, tables of values, rates of change, and linear equations.

Learning Expectations

AA1. develop and explore a variety of social-emotional learning skills in a context that supports and reflects their learning in connection with the expectations across all other strands.

- recognizing sources of stress that present challenges to mathematical learning
- building healthy relationships and communicating effectively in mathematics
- developing critical and creative mathematical thinking

A1. apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning

- problem solving
- representing

A2. make connections between mathematics and various knowledge systems, their lived experiences, and various real-life applications of mathematics, including careers.

B3. apply an understanding of rational numbers, ratios, rates, percentages, and proportions, in various mathematical contexts, and to solve problems

> **B3.1** apply an understanding of integers to describe location, direction, amount, and changes in any of these, in various contexts

B3.2 apply an understanding of unit fractions and their relationship to other fractional amounts, in various contexts, including the use of measuring tools
B3.3 apply an understanding of integers to explain the effects that positive and negative signs have on the values of ratios, rates, fractions, and decimals, in various contexts
B3.4 solve problems involving operations with positive and negative fractions and mixed numbers, including problems involving formulas, measurements, and linear relations, using technology when appropriate

C1. demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations
C1.2 create algebraic expressions to generalize relationships expressed in words, numbers, and visual representations, in various contexts
C1.3 compare algebraic expressions using concrete, numerical, graphical, and algebraic methods to identify those that are equivalent, and justify their choices
C1.4 simplify algebraic expressions by applying properties of operations of numbers, using various representations and tools, in different contexts
C1.5 create and solve equations for various contexts, and verify their solutions

C3. represent and compare linear and non-linear relations that model real-life situations, and use these representations to make predictions
C3.2 represent linear relations using concrete materials, tables of values, graphs, and equations, and make connections between the various representations to demonstrate an understanding of rates of change and initial values
C3.3 compare two linear relations of the form $y = ax + b$ graphically and algebraically, and interpret the meaning of their point of intersection in terms of a given context

D2. apply the process of mathematical modelling, using data and mathematical concepts from other strands, to represent, analyse, make predictions, and provide insight into real-life situations
D2.2 identify a question of interest requiring the collection and analysis of data, and identify the information needed to answer the question

Cross Curricular Connections

Physical education (active living)
Science (conducting an experiment, collecting data, analyzing data)

Learning Goals and Success Criteria:

LG1: We are learning about the connections between linear relations, tables of values, and graphs.

SC1: I can represent a linear relationship using a table of values.
SC2: I can represent a linear relationship using a graph.

LG2: We are learning to make connections between the various representations of a linear relationship to demonstrate an understanding of rates of change and initial values in order to determine the equation of the line and make predictions by solving equations.

SC1: I can identify the rate of change of a linear relationship.
SC2: I can identify the y-intercept of the graph of a linear relationship.
SC3: I can compute the slope of the graph of a line.
SC4: I can create an equation to represent a linear relationship.
SC5: I can predict values not on the graph of a line by solving an equation.

LG3: We are learning to identify and interpret the point where two lines meet.

SC1: I can graph two linear relations on the same grid.
SC2: I can determine the point of intersection of the two lines graphically.
SC3: I can interpret the meaning of the point of intersection of two lines.

CONSIDERATIONS THROUGHOUT THE LESSON

Differentiated Instruction and Universal Design for Learning

Use Visibly Random Groupings to create small groups.
Use flexible small group instruction for students who need support.
Consider extensions for groups that finish early.
Preview vocabulary: table of values, scatter plot, positive relationship, negative relationship, scale on a graph, initial value, rate of change, dependent variable, independent variable
Scaffolding (See teacher prompts embedded in the Action section.)

Assessment

Listen for understanding of how students determine a table of values, create graphs and form equations of linear relationships.

Are students seeing the connections between the numeric, graphical and algebraic representations of a relationship?

During the Action, circulate while students are working on the questions. Note on your tracking tool what expectations you observe are being met, student by student. Have conversations with students to assess their Thinking (Problem Solving mathematical process) and Communication (Representing mathematical process). Use these observations and conversations to sequence the sharing of student work.

Students will self evaluate using the Success Criteria.
Students will individually complete the Desmos activity (see Consolidation section).
Teacher will provide descriptive feedback during the Desmos activity.

RESOURCES AND LEARNING ENVIRONMENT

Educator Resources Needed

Computer with Internet and data projector
Vertical whiteboards or chart paper
Access to videos: MOST JUMPING JACKS IN ONE MINUTE, Candle Clock - Candle Timer

Student Materials Needed

Computer with Internet
Graph paper
Dry erase markers/markers
Stopwatch or other timing device
Table of values (illustrated in Action)
Handout
Desmos activity

Learning Environment Considerations

The Minds-On section starts with students working in groups for the video activities for both part 1 and part 2.

In the Action section, students will work in pairs to collect data for the jumping jack activity (part 1). Each group will need space to work with a vertical non-permanent surface or chart paper. They should be able to compare ideas with other groups.

The Consolidation will be a whole class activity followed by an individual assessment activity in Desmos.

LESSON CONTENT

Minds-On (Part 1: 10 Minutes; Part 2: 5 Minutes)

Part 1:
Note to teacher: Play the following Jumping Jacks video, beginning at 0:29

Show video.
Ask the students what they notice and what they wonder. Ask students how many jumping jacks they think they can do in 1 minute. How many jumping jacks could they do in 1 hour? Tell the students that the world record, from 09 May 2020, is held by Mario Silvestri of Italy. He completed 3873 jumping jacks in 1 hour.

(**Note to teacher**: It may be optimal to move to Part 1 of the Action now, then come back to Part 2 of the Minds-On before beginning Part 2 of the Action.)

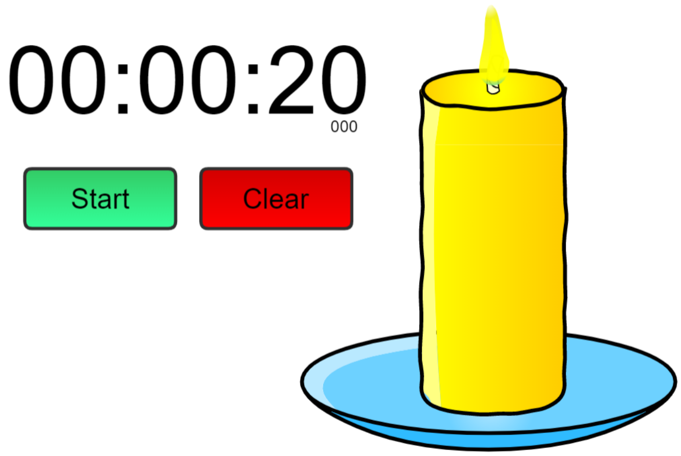
Part 2:
Play the following video and set the candle timer for 10 to 20 seconds.

Candle Clock - Candle Timer

Ask students what they notice and what they wonder as they watch (as many times as needed).

Possible responses:

- I notice that the candle is getting shorter.
- I notice that the time is going down.
- I notice that the time is very precise.
- I wonder whether the flame will last until the end of the timer.
- I wonder what would happen if the timer was set to 30 seconds?



Action (300 Minutes)

Part 1

Students should work in pairs to collect data for time vs. number of jumping jacks. One student should do jumping jacks while counting, while their partner notes and records the time on a stopwatch or other timing device every 5 jumping jacks. Then they should trade places. Each student should complete 30 jumping jacks. They should complete a table of values similar to the one shown below. The goal is for students to try and keep a consistent pace to create a linear relationship between number of jumping jacks and time in seconds.

Number of Jumping Jacks	Time (s)
0	
5	
10	
15	
20	
25	
30	

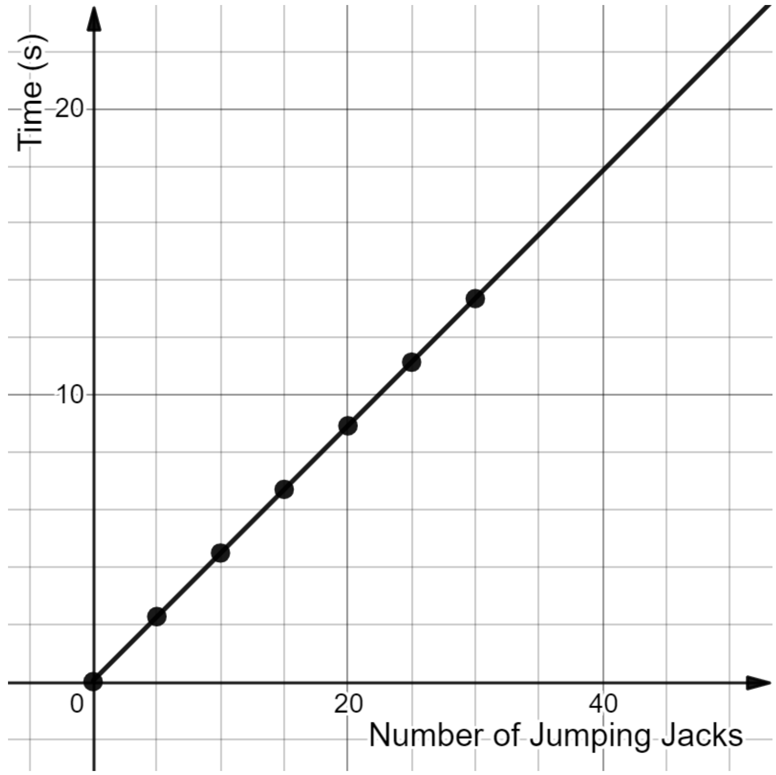
Students, in pairs, should collect and graph their data. Students will then calculate their rate of change and determine the equation of their relationships.

Sample data: Student 1

Table of values:

Number of Jumping Jacks	Change in Number of Jumping Jacks	Time (s)	Change in Time
0	---	0	---
5	5	2.27	2.22
10	5	4.49	2.22
15	5	6.71	2.22
20	5	8.93	2.22
25	5	11.15	2.22
30	5	13.37	2.22

Graph:



Calculating rate of change (RoC) using rise over run between any 2 consecutive points:

$$\text{RoC} = \frac{2.22}{5}$$

RoC = 0.444

y = 0.444x

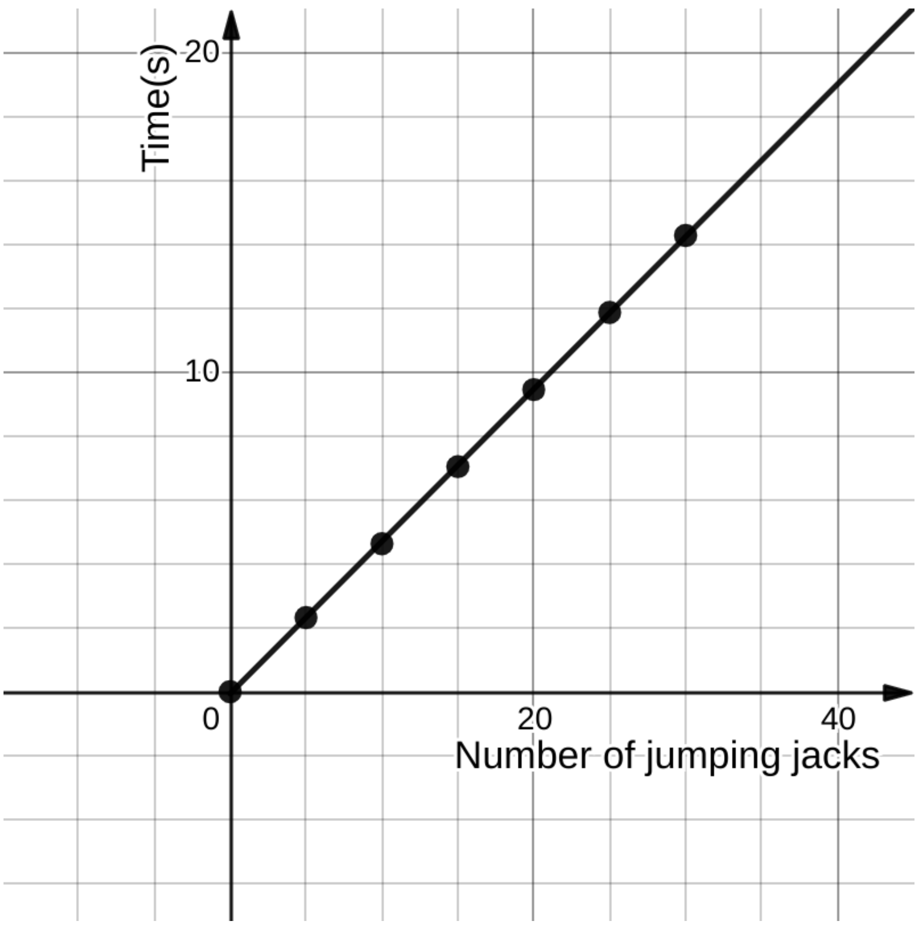
Sample data: Student 2

Table of values:

Number of Jumping Jacks	Change in Number of Jumping Jacks	Time (s)	Change in Time
0	---	0	---
5	5	2.32	2.41

10	5	4.63	2.41
15	5	7.04	2.41
20	5	9.45	2.41
25	5	11.86	2.41
30	5	14.27	2.41

Graph:



$$RoC = \frac{2.41}{5}$$

RoC = 0.482

y = 0.482x

Once this has been completed, students should consider the following question:

Question 2: How many jumping jacks would you do in **an hour**?

Students should consult with their partner and together they will develop a strategy for answering this question. Circulate and observe what strategy or strategies students are developing. Note each different strategy. Also listen for students who are unable to come up with a strategy. Try to determine what the issue is -- is it determining how many seconds are in a minute, or is it how to extend the pattern to larger numbers? Offer these prompts accordingly: ""What are we looking for?", What do we know?", "How can we use what we know to determine what we are looking for?", "How many seconds are there in a minute?", "How many seconds are there in an hour?"

Take a few moments to share some strategies you have observed with the whole class before giving the student pairs the next two questions (3 and 4).

Question 3: Suppose you forgot to start the timer when you started doing jumping jacks. How will this delay in starting the timer affect the graph? How will this affect the equation?

Again, students should work with their partner and agree on their answer (or, if consensus cannot be reached, each should be willing to support their own perspective). Teacher prompts: What two things are we measuring? Which of these would be affected, do you think? Is there any data point that will not change? What would change in your table of values? What would stay the same?

A mini lesson on initial values may be useful at this point.

Question 4: A student has a delay of 10 seconds before he starts doing jumping jacks and then he does one jumping jack every 2.5 seconds. How long will it take him to do 73 jumping jacks? If he stopped after 5 minutes, how many jumping jacks did he do?

Teacher prompts: What tells you that this is a linear relationship? How would you create a table of values for this situation? What information relates to the rate of change of this relationship? What equation would represent this situation? How can the equation help you determine the answer you need?

Once again, while the students are working with their partners on these two questions, circulate and make note of the different strategies students are using.

Part 2 (If you have postponed sharing Part 2 of the Minds-On, go back to it now.)

Conduct a new visibly random grouping of students to create groups of 3. Each group will answer the following questions, one by one.

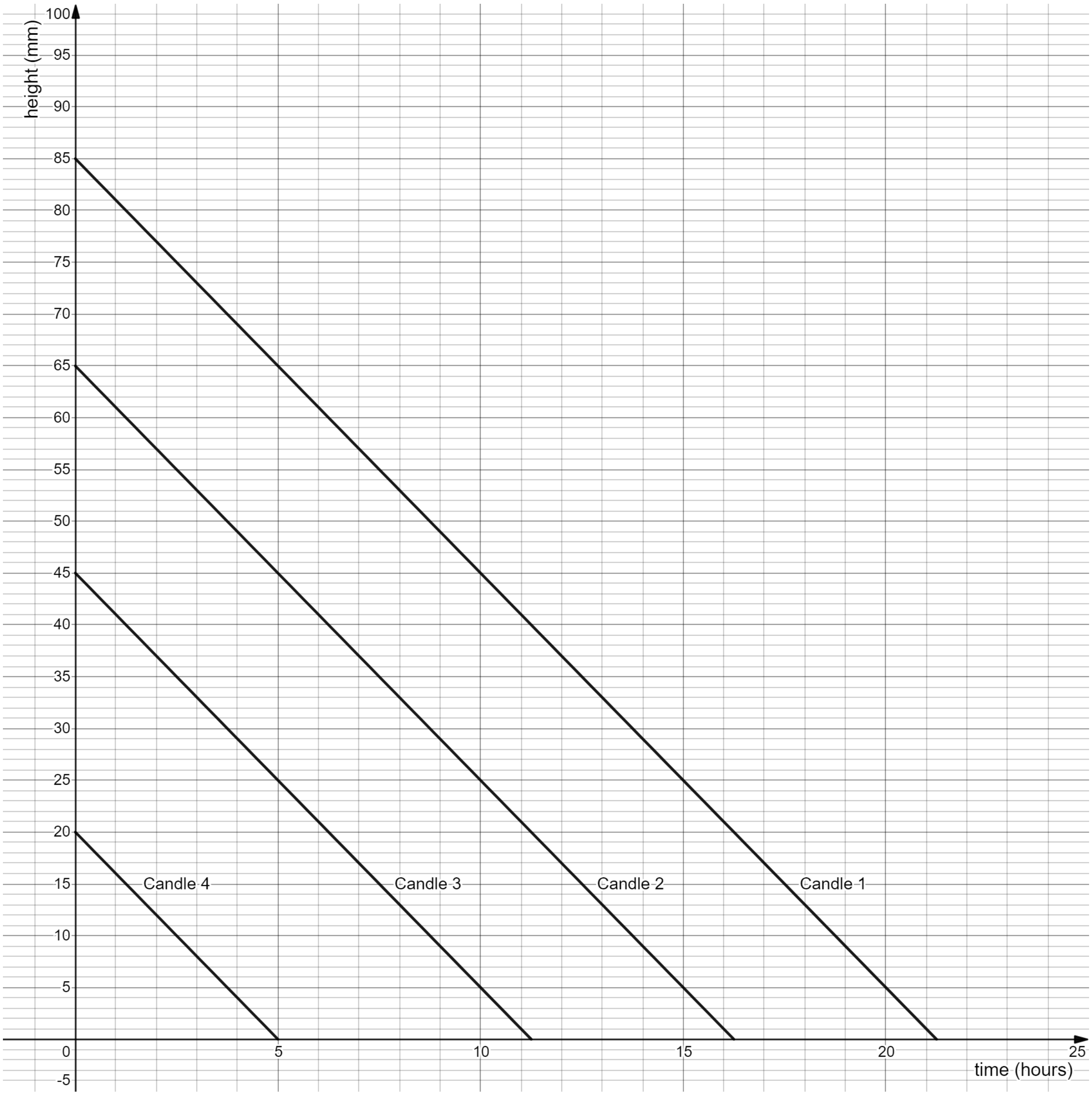
Question 1: If each of the four candles shown below burn at the same rate of 4 mm of height per hour, create a table of values that shows the height of each of the candles at 1 hour, 2 hours, and 3 hours. Sketch the height (mm) vs. elapsed time (hours) graph for all four candles on the same grid. The initial heights are provided in the table below.

Candle	Height (mm)
1	85
2	65
3	45
4	20

Note to teachers: This task will allow students to revisit the concepts of rate of change and initial value within a new context. This provides an excellent opportunity for teachers to assess where each student is in their understanding of these two concepts and how they are connected to change in a real life context. In this part of the Action, students will be making new connections between the initial value of a linear relationship and the y-intercept of the graphical representation of that relationship, as well as the rate of change of the relationship and the slope of the line. They will observe that different linear relationships that share the same constant rate of change will create parallel lines when graphed.

Connections may be made here to vertical transformations of lines if that has been taught. Students may be asked to connect the vertical translation required to move from one line to a second line with the information provided about the initial height of the candles. If transformations have not yet been introduced, returning to this lesson could serve as Minds-On to the new learning.

Sample graph:



As students are working in their groups, circulate and note any difficulties they may be having.

Review the 4 graphs with the whole class, ensuring that each group has completed the task successfully. Then ask the next question:

“These 4 lines are parallel. What do you know about the candles burning that might allow you to predict that the lines would be parallel?”

Ask that students just think about their answers quietly for at least 30 seconds. Have them share their thinking within their group for 2 - 3 minutes. Walk around to get a sense of the thinking of a few groups. Then accept their responses. (Possible responses: We know that the rates at which the 4 candles are burning are the same, so they will lose the same amount of height in the same length of time. However, we also know that they start at different heights, so the lines are all different.)

Reflection for teachers regarding assessment: Are all students able to explain why equal rates of change produce parallel lines? Does every student make the connection between the initial height of the candle and the y-intercept of the graph, even if they are not using precise language yet?

Have the students work with their partners on the next 3 questions. As they discuss their strategies and share their perspectives, listen for their reasoning. Record what you observe in your assessment tracking sheet.

Question 2: How would you use the graph to determine how long each candle will burn?

Question 3: How could you use the table of values to determine how long each candle will burn?

Question 4: If you didn’t have the graphs or the tables of values yet, which method do you feel is more efficient? Why? (Assure students that this is asking for their opinion, not the “right answer”. Listen to students describe the process they need to go through to answer this question these two ways.)

Bring the class together so they can share their answers to these three questions. Ensure that all voices are respected as students share their thinking, particularly when sharing their perspectives for Question 4.

Once each of the 3 questions have been discussed, ask that they think about what other way(s) we could verify our answers. If necessary, prompt students to offer “use an equation” as a new method.

Assign one candle to each group of three and ask that they determine the equation that represents their candle’s graph. They are to use their equation to verify their answer to the question “How long will your candle burn?” Anticipate that some students will substitute height = 0 and solve for time, while others will substitute height = 0 and the time they found using the graph/table of values and verify that their equation balances using “left side equals right side” (LS = RS). Before going on to the next section, be sure that students recognize that either method can be used.

Answers:

Candle 1: 21.25 hours

Candle 2: 16.25 hours

Candle 3: 11.25 hours

Candle 4: 5 hours

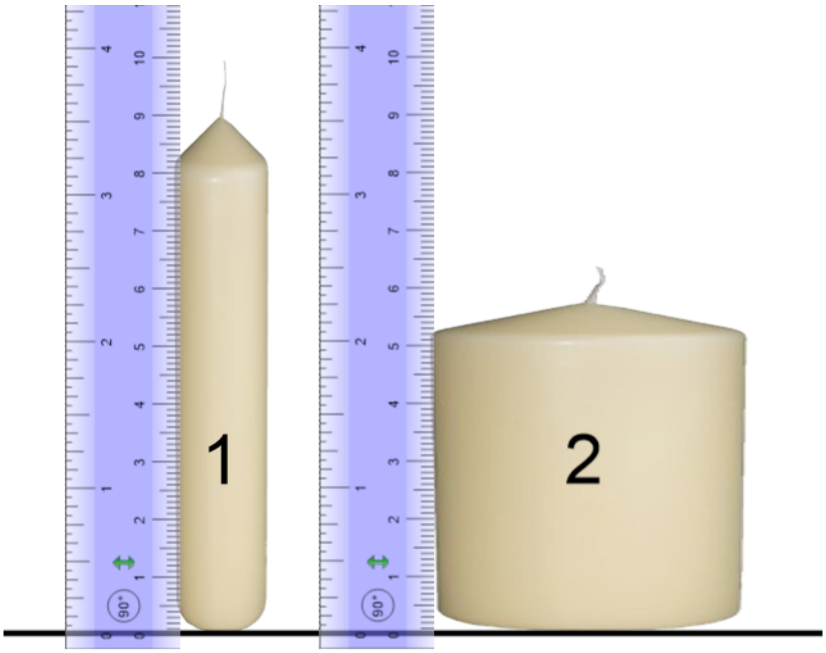
Provide each group of three students with the handout. Asking them to look at only the first image of the two candles side-by-side, have a discussion about what factors they think would affect the rate at which the height of the candle reduces. Ask which of these two candles they think will have a **greater** rate of change and why they think that. You may also want to ask and discuss which of these two candles will have a **greater number** representing the rate of change. (It is helpful to reinforce that, for example, -4 is less than -2 even though 4 is larger than 2, and that numbers get larger as we move right on the number line. However, when we are talking about “greater rate of change”, we are looking for the larger amount of change (so when graphing a line, we are looking at how steep the line is, regardless of whether it is going up to the right of down to the right), but not if the change is positive or negative. In cases like this, “greater” depends on the context: are we talking about the number itself or how quickly the candle is burning down?)

Question 5: After how much time will the two candles shown have the same height?

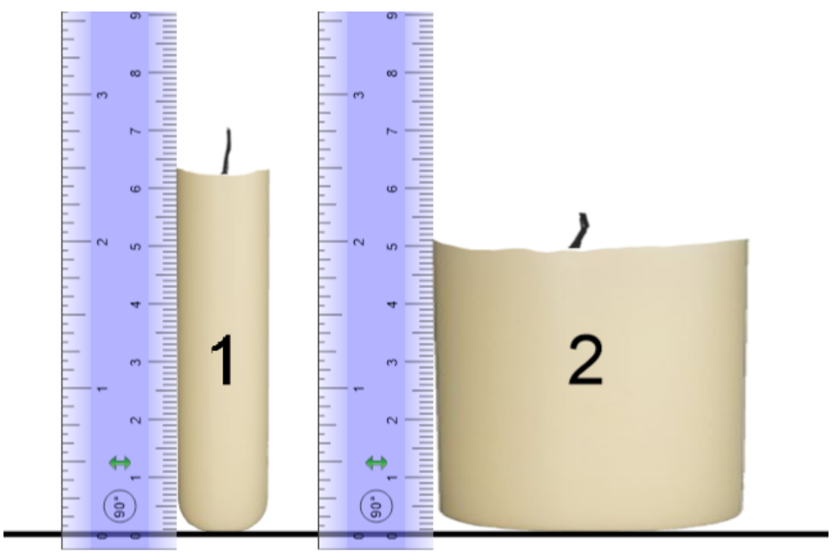
Ask students what information they need to answer this. You may want to have a discussion about assumptions and agree to a set of assumptions before sending students back to their groups to solve this problem. (For example: Do we use the wick when we measure height? (It also needs to burn down, but it won’t burn at the same rate as the candle. Do we include the “pointy part” of the candle when we measure the height? (This also will probably burn away faster than the thick part of the candle.) Conclusion: Probably we shouldn’t start measuring until we get to the thick part of the candle so that our rate of change data is valid.)

Ask students to graph the heights of both candles over time on the same grid. Students will be solving this problem by graphing.

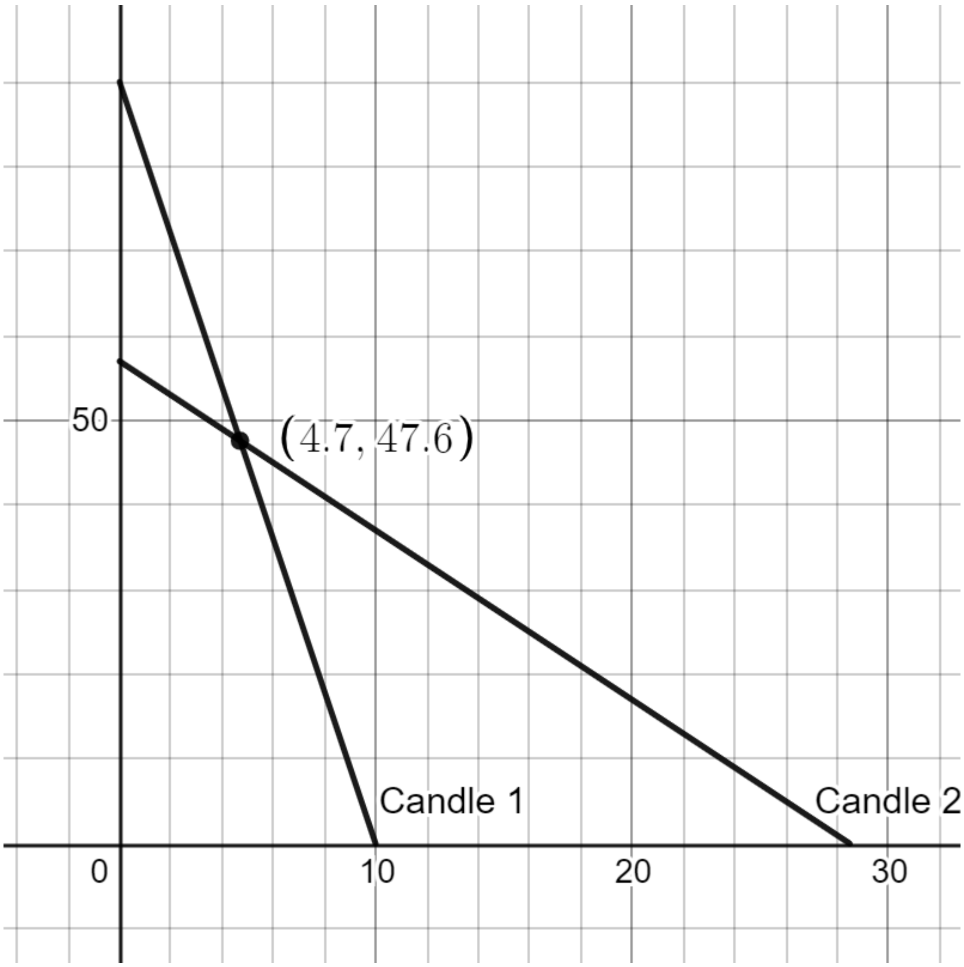
Two images are provided on the student handout. The first allows students to determine the initial heights of the candles (after the discussion about assumptions that must be made). The second allows students to determine the rate of change of height over time for both candles.



After 3 hours:



Possible solution



Again, circulate as students are working. Check that students are able to fairly accurately determine the initial heights of the two candles as well as their burning rates. Observe student work and select some ideas to share with the class once the groups have completed this task.

Bring the class together as a whole. Collect student responses. Talk about why answers might vary (It was hard to tell exactly how tall each candle was at the start; it was hard to tell exactly how tall each candle was after 3 hours; it was hard to read the exact point where the two lines crossed. Discuss how important it is that people (scientists, geographers, medical researchers, etc) use very accurate measurements when they are creating a mathematical model.

Lead the class in a discussion of sets of circumstances in which you might want to know the moment when two quantities, that start at different values and change at different rates, attain the same value. Encourage students to consider situations from their own lives. (If students are struggling to come up with ideas, some prompts could be: You pass a sign on the highway that indicates the distance left on the highway until the next gas bar (a constant function). You check the range indicator on your car (and we can assume that the car is equipped with cruise control so range has a linearly declining relation with time) - will you make it? (Note that this scenario imagines a snapshot in time, but it could be rethought as the distance remaining declines while the distance travelled increases over the same time interval.); a baby born prematurely often has very low weight, but they frequently gain weight at a greater rate than full-term babies. What question(s) might you answer using intersecting lines?)

Extension Opportunities

Students can determine the equation represented by the following data sets:

x	y
3	10
5	16
7	22
9	28

x	y
2	19/3
5	25/3
8	31/3
11	37/3

x	y
3	9

6	18
7	21
12	36

Additional data sets can be created which could include negative x-values.

Students can interpolate or extrapolate values based on their equations to further practice solving equations.

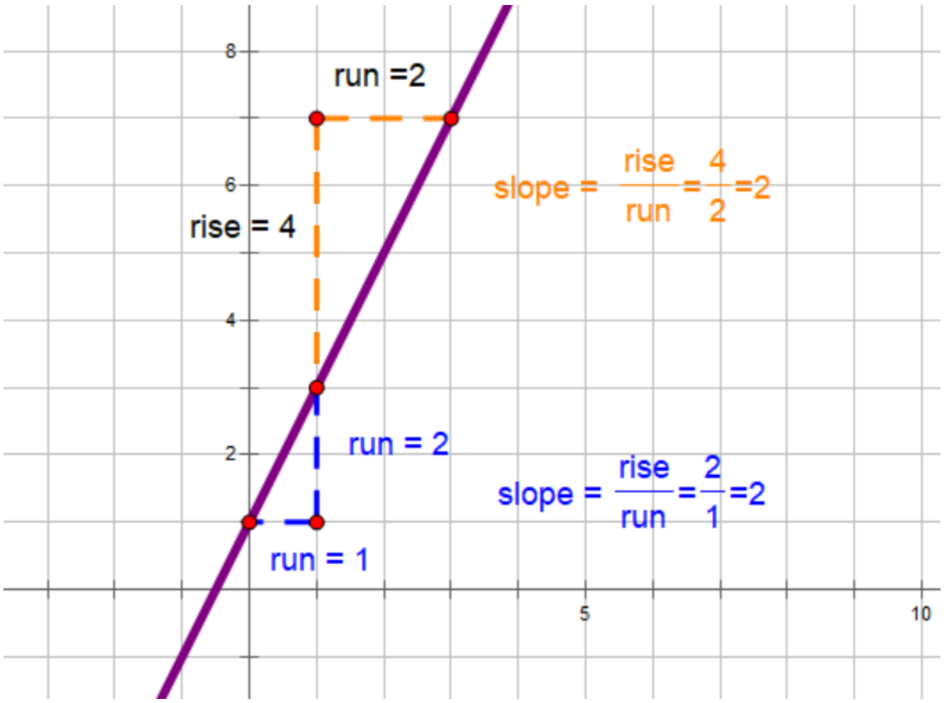
Consolidation (75 Minutes)

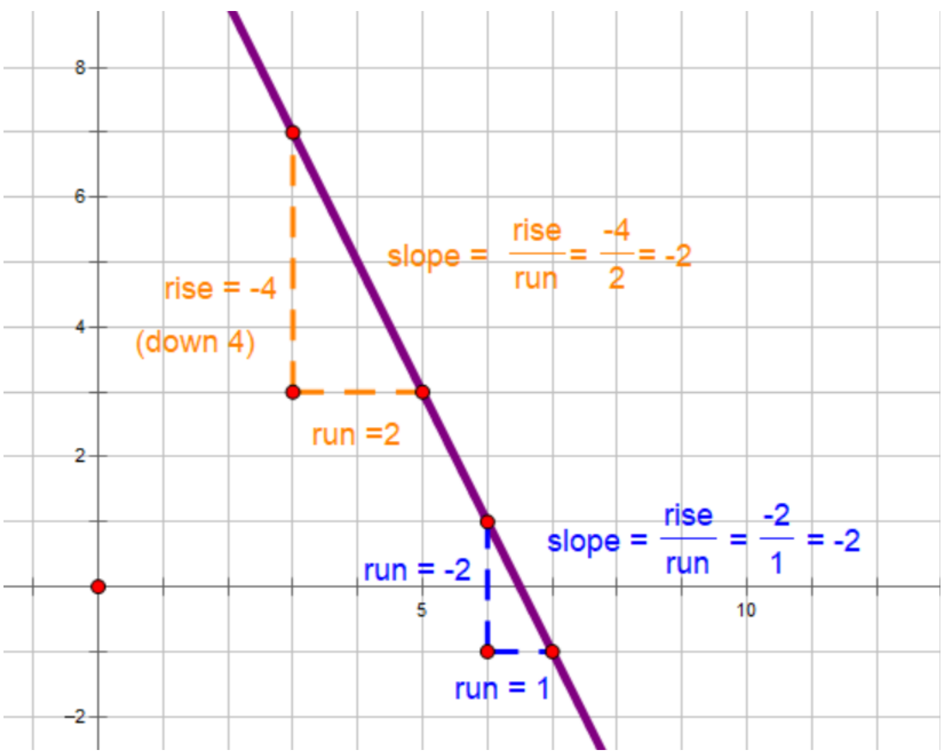
During the Consolidation, the teacher will review the concepts explored during the two tasks in the Action. The principal goals will be to ensure that connections outlined in Learning Goal 2 are solidified:

The slope of a line corresponds to the rate of change of a linear relationship. The slope of a line represents the constant rate of change of the relationship.
We can find the slope of a line by computing the rise (the vertical distance as we move from a first point to a second point), the run (the horizontal distance as we move from that first point to that second point), and then dividing the rise by the run.

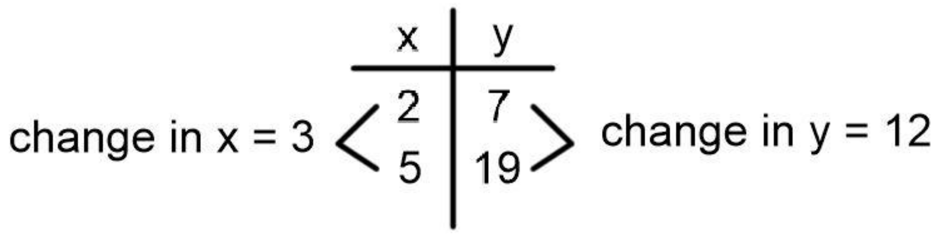
We often use the letter “a” to represent the slope when we are writing the equation of a line in its general “slope, y-intercept form”: $y = ax + b$.

From a graph:


run =2
slope = rise/run = 4/2 =2
rise = 4
run = 2
slope = rise/run = 2/1 =2
run = 1
8
6
4
2
5
10


slope = rise/run = -4/2 = -2
rise = -4
(down 4)
run =2
slope = rise/run = -2/1 = -2
run = -2
run = 1
8
6
4
2
-2
5
10

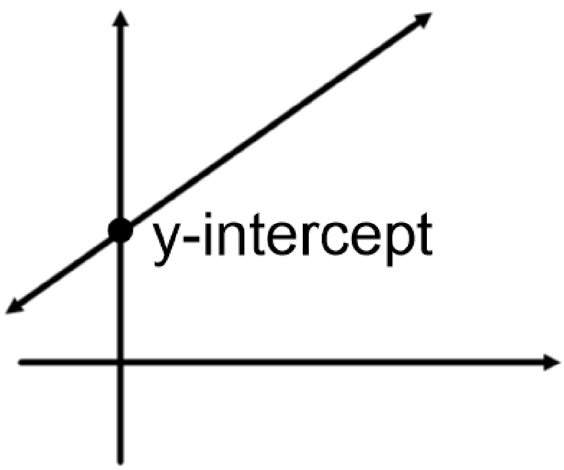
From a table of values:



$$a = \frac{12}{3}$$

$$a = 4$$

The y-intercept of the graph of a linear relationship corresponds to the initial value of that relationship. It is equal to the value of the dependent variable when the value of the independent variable is 0. The y-intercept occurs where the graph (line) crosses the vertical axis (which is where the value of the independent variable is 0). We often use the letter “b” to represent the y-intercept when we are writing the equation of a line in its general “slope, y-intercept form”: y = ax + b.



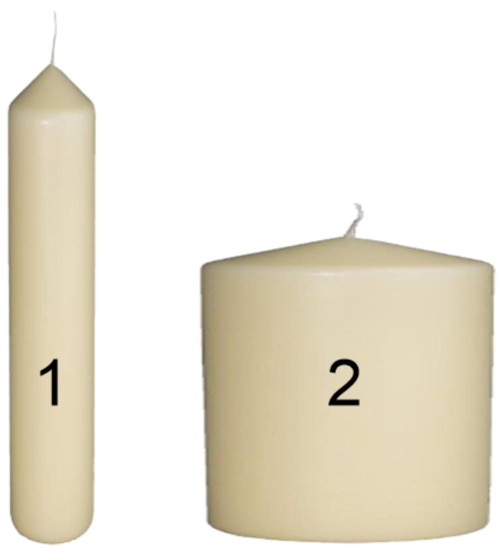
Students should be provided with an opportunity for further practice identifying slopes and y-intercepts of the graphs of linear relationships when given descriptions of the relationship, when given a table of values, and when given a graph.

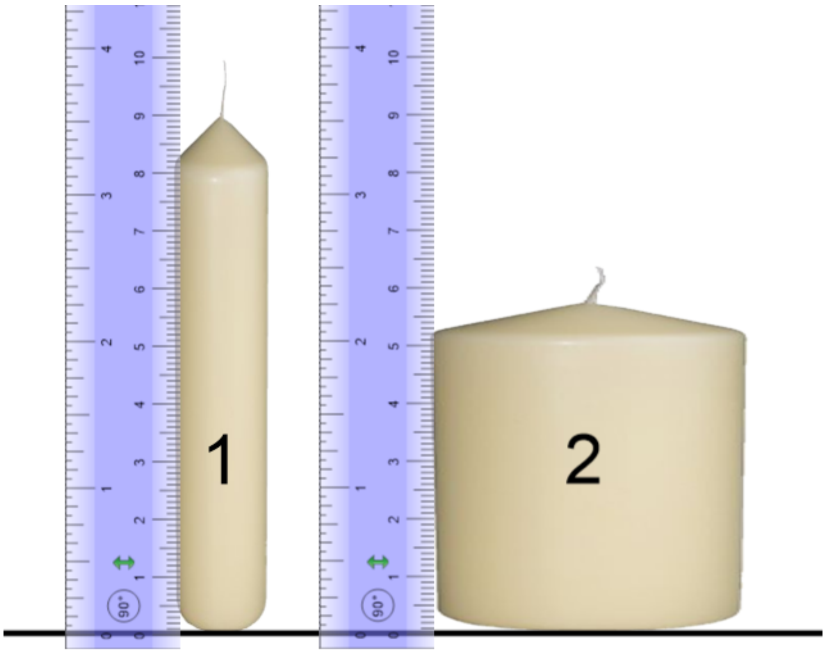
Students can work individually on the Desmos activity as a further consolidation of learning. The teacher may use this activity for assessment purposes. (The teacher should monitor the dashboard in Desmos and provide descriptive feedback as needed.)

Appendix:

Student handout for Question 5

Question 5: After how much time will the two candles shown have the same height?





Height after 3 hours of burning: