



Save Time... Use Code

Task Description

Prior Learning and Placement of Task during Semester

Diagnostic Task & Look fors

The Task

Teacher Notes

Assessment

Additional Resources



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Task Description Students will explore relationships between quantities/measurements and look for different ways to represent relationships and algebraic thinking.	Big Idea <ul style="list-style-type: none">Generalized relationships come from prior understanding.Technology is a powerful tool for repetitive processes.Algebraic thinking connects these two ideas.
Students will take advantage of technology to thoroughly explore different relationships.	Duration Diagnostic: about 20 minutes Task: minimum 130 minutes

Recommended Materials <ul style="list-style-type: none">A device or Chromebook for each student (phones are not recommended)Access to any of the following applications: Google Sheets Desmos (free web based program, no sign in required; Students can create an account to save their work) Scratch (free web based program, no sign in required; Students can create an account to save their work)

Learning Goal(s) <ul style="list-style-type: none">Students will create algebraic expressions to generalize a relationshipStudents will evaluate expressions to collect/generate dataStudents will create code to represent a mathematical relationshipStudents will explore the properties of a non-linear relationship Sample Success Criteria <ul style="list-style-type: none">I can pose and solve problemsI can identify a relationship between two or more quantitiesI can use a relationship to find new informationI can express a relationship in words, an expressionI can use code to translate my algebraic thinkingI can interpret the results of my code and refine as neededI can identify some of the properties of a non-linear relationship
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Overall Expectations AA1. Social-Emotional Learning Skills develop and explore a variety of social-emotional learning skills in a context that supports and reflects this learning in connection with the expectations across all other strands A1. Mathematical Processes apply the mathematical processes to develop a conceptual understanding of, and procedural fluency with, the mathematics they are learning C1. Algebraic Expressions and Equations: demonstrate an understanding of the development and use of algebraic concepts and of their connection to numbers, using various tools and representations C2. Coding: apply coding skills to represent mathematical concepts and relationships dynamically, and to solve problems, in algebra and across the other strands C4. Characteristics of Relations: demonstrate an understanding of the characteristics of various representations of linear and non-linear relations, using tools, including coding when appropriate See teacher notes for possibility of including B1 and E1



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Social-Emotional Learning (SEL) Skills

- Recognizing a range of emotions in self and others
- Recognizing that new or challenging learning may involve a sense of excitement or an initial sense of discomfort
- Using an iterative approach to solve a problem, including reframing questions, trying out different methods, estimating, and guessing and checking solutions
- Embracing mistakes as a necessary and helpful part of learning
- Making connections
- Developing Critical and Creative Mathematical Thinking

** This overall expectation is to be included in classroom instruction, but not in assessment, evaluation, or reporting. See [further information](#) about approaches to instruction that support all students as they work to apply mathematical thinking, make connections, and develop a healthy identity as mathematics learners to foster well-being and the ability to learn mathematics.*

Mathematical Processes

There are opportunities for students to engage in the [mathematical processes](#) throughout this task.

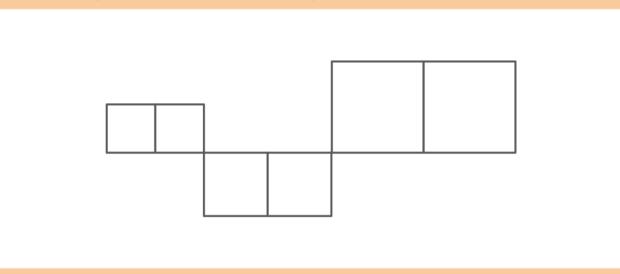
For example:

- **Problem Solving:** critical thinking, math identity, knowledge, collaboration, lived reality, creative thinking, confidence
- **Reasoning and Proving:** justification, proportional reasoning, algebraic reasoning, spatial reasoning, numbers, operations, geometric properties, measurement
- **Reflecting:** identify what is working, what isn't working, appropriate strategy, reasonableness of their answer
- **Connecting:** connect different mathematical concepts
- **Communicating:** share ideas, understandings and solutions, provide feedback, pose questions
- **Representing:** represent math relationships using pictures, diagrams, numbers and symbols
- **Selecting Tools and Strategies:** test, revise, confirm reasoning, remembering how they solved a problem

Prior Learning and Placement of Task during Semester

This task could be done at any point, but students need to know how to find the area of a rectangle (square). See [teacher notes](#) for the way in which the task may unfold depending on the prior learning of your students.

Diagnostic Task & Look fors

Diagnostic Slides for the diagnostic task can be found here .	Look Fors
What do you notice? What do you wonder? 	<p>The series of prompts for the first image are to get students thinking about relationships between unknowns.</p> <p>In this case, students are given information in stages (a slow reveal) to encourage them to consider how what they see changes with new information.</p> <p>Some sample solutions of finding areas can be found here.</p>

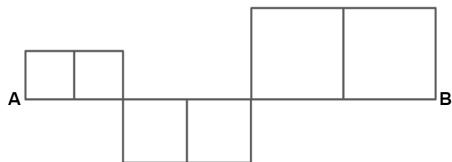


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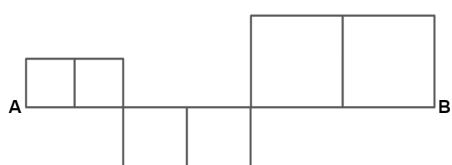
The figure is made of 6 squares.

The length of AB is 28.

What are some questions we could ask?



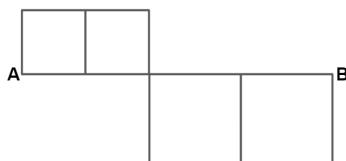
Given that the diagram is not drawn to scale,
find a solution to as many of your questions as possible.



Task

(The task is also located in [this slide deck](#).)

Given the length of AB is 22 cm,
what are the possible areas?



Students may enter this problem from paper/pencil, using tables in sheets or with coding (Sheets or Scratch). This may also be a continuum of where students could go with the task.

Some sample solutions of finding areas can be found [here](#).

Solution using [Desmos with area sliders](#).

Task extension

The length of AB is 28.

Adjust your code to determine the possible areas of the 6 squares.





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Teacher Notes

Teacher moves: As students are generating possible areas, ask, “Are there any other possible areas? How do you know?”

The goal here is to present a situation in which using a code will aid in both representing student thinking and to generate all possible areas.

At this point, students can be presented with options moving forward to be able to represent their algebraic thinking. The choice ideally will be driven by the student.

Using Google Sheets

- [Sample template for students to use](#)
- [Sample solution and teacher notes](#)

Using Scratch

- [Sample solution in Scratch](#)
- Students could also examine already written code to modify it to suit their needs
 - [Remix and Modify Option 1 in Scratch](#) (with prompts & hints)
 - [Remix and Modify Option 2 in Scratch](#) (no prompts)
- More support for a Scratch Solution can be found in this [slide deck and speaker notes](#)

Once students have been able to generate more possible outcomes in a more systematic way, a non-linear relationship can be explored in both a table and in a graph. This is more readily done if students are being systematic in finding the possible areas, which is where using google sheets or creating a code will be beneficial.

Possible directions this task may go:

- Number sets in understanding the possible values of each square length could be any real number, not just whole numbers.
 - Finding the optimal (minimum) area of the figure could be examined in this task. This would lead to a discussion of number sets, as students may likely have only whole numbers as side lengths. What numbers are between 5 and 6, could start a conversation about different number sets (descrete vs continuous), as well as density and limits (B1)
- Measurement may come out - how changing one variable affects the other
 - As students are building their table of values, they may start to see the relationship of changing one side length to other side length and/or area of the square or whole figure (GM1)

Assessment

Self-Reflection:

Refer back to the success criteria for this task:

- I can pose and solve problems
- I can identify a relationship between two or more quantities
- I can use a relationship to find new information
- I can express a relationship in words, an expression
- I can use code to translate my algebraic thinking
- I can interpret the results of my code and refine as needed
- I can identify some of the properties of a non-linear relationship

Provide evidence of how you have met the criteria.

This task can be included in the [Student Portfolio of Process Expectations \(Google Slides\)](#)



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Additional Resources

Introduction to Scratch - See tutorials in Scratch

[Getting Started with Scratch](#)

Access: <https://scratch.mit.edu/>

Google Sheets Tutorials

[Absolute and Relative Cell References in Google Sheets](#)

[Creating Complex Formulas in Google Sheets](#)

[Creating Charts in Google Sheets](#)

What Works? Research into Practice: Research monograph # 69

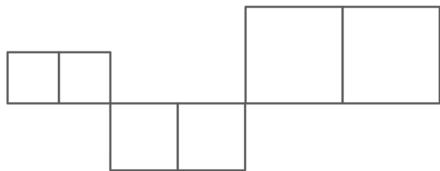
[Computer Coding in the K-8 Mathematics Curriculum?](#)



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Diagnostic Task Possible Student responses

What do you notice? What do you wonder?



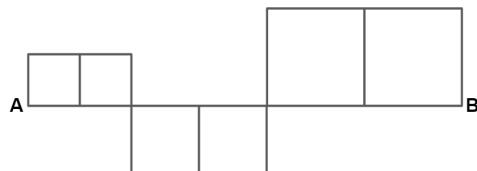
- There are six squares along a line
- There are three pairs of squares
- There are two of each kind of square
- The pairs of squares are different sizes
- The pairs of squares grow from left to right

Teacher note: The hope is that students will see squares in this diagram, and begin to wonder about relationships between the squares (i.e. The small square look like half the length of the large squares). This latter point may be more prevalent with the next prompt/added information.

The figure is made of 6 squares.

The length of AB is 28.

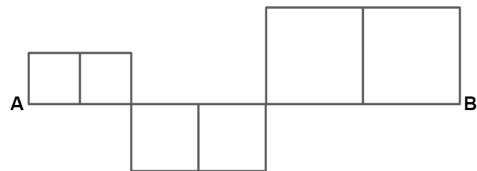
What are some questions we could ask?



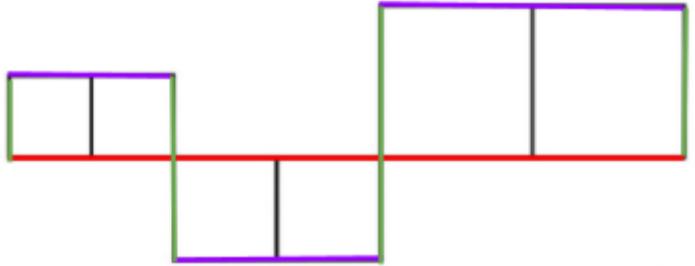
- What are the lengths of the sides?
- What is the total length of all segments?
- What is the perimeter of the shape?
- What is the area of the shape?
- How many of the smallest squares go into the largest square?

Teacher note: With the added information of the length of AB, the hope is students will begin to consider the lengths of the other sides.

Given that the diagram is not drawn to scale, find a solution to as many of your questions as possible.



Finding perimeter of the shape:



28 cm
total = 28 cm
total = 28 cm

$$P = 3 \times 28 \text{ cm}$$

Finding total length of all segments:



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— 28 cm

— total = 28 cm

— total = 28 cm

— total = half of 28 cm

Other relationships, expressed algebraically but also in words:

- Try numbers so the length is 28 (1cm, 5cm, 8cm)
- One of each side from each square sums to 14
- $2x + 2y + 2z = 28$
- $x + y + z = 14$

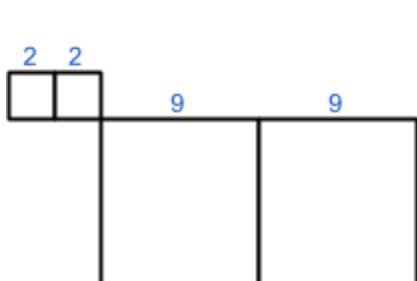


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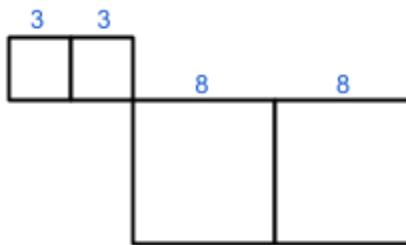
Sample Solution - Paper/Pencil

Solution 1:

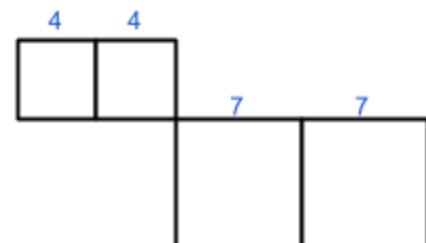
Given the length of AB is 22 cm,
what are the possible areas?



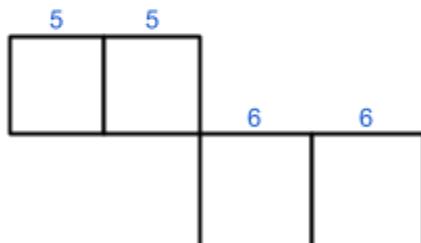
Area = 170



Area = 146



Area = 130



Area = 122



Area = 242

Are there any other possible areas? How do you know?

Is a length of zero possible with the image?



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Solution 2:

Given the length of AB is 22 cm,
what are the possible areas?



Let a represent the side lengths of the first 2 squares.

Let b represent the side lengths of the second 2 squares.

$$2a + 2b = 22$$

$$2a = 22 - 2b$$

$$a = 11 - b$$

a	b	Area
0	11	242
1	10	202
2	9	170
3	8	146
4	7	130
5	6	122
6	5	122
7	4	130
8	3	146
9	2	170
10	1	202
11	0	242

$$x_1$$

$$2x_1^2 + 2(11 -$$

$$0) = 242$$

$$1) = 202$$

$$2) = 170$$

$$3) = 146$$

$$4) = 130$$

$$5) = 122$$

$$6) = 122$$

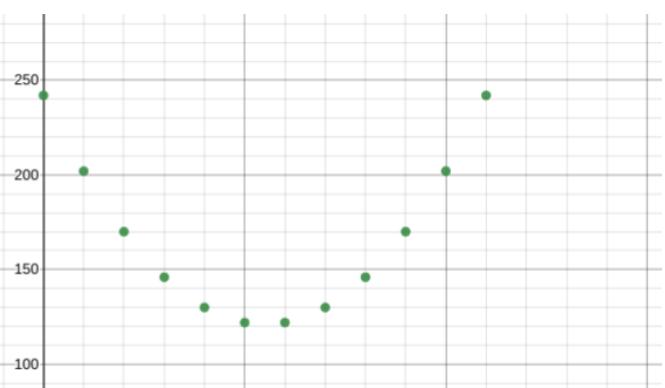
$$7) = 130$$

$$8) = 146$$

$$9) = 170$$

$$10) = 202$$

x

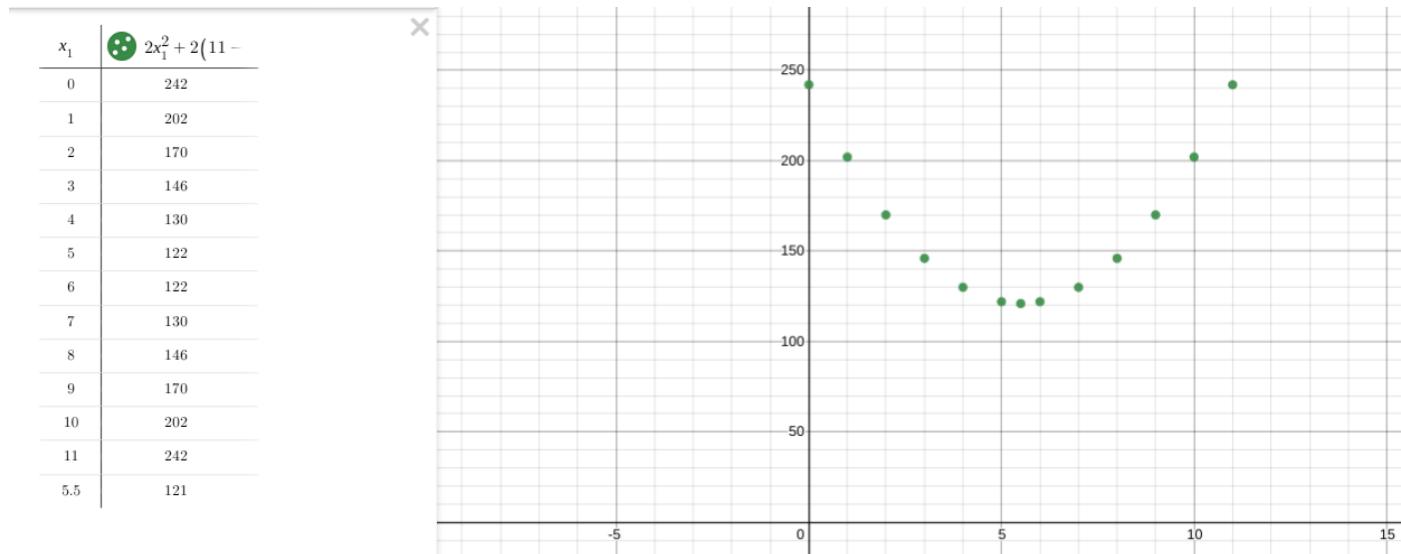




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If a and b do not need to be whole numbers the area may be smaller when $5 < a < 6$

a	b	Area
5	6	122
5.4	5.6	121.04
5.45	5.55	121.01
5.5	5.5	121
5.55	5.45	121.01
5.6	5.4	121.04
6	5	122



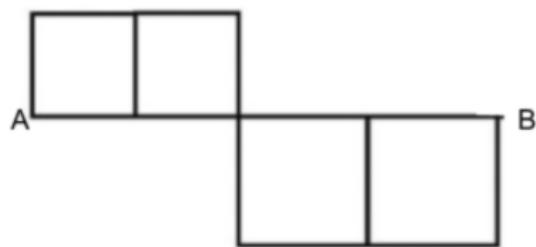


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Google Sheets sample solution

C2 ▾ | fx | $=(\$A\$2-(2*B2))/2$

	A	B	C	D	E	F	G	H	I	J	K
1	LineLengthAB	Side1	Side2	Total Area							
2	22	0	11	242							
3		0.5	10.5	221							
4		1	10	202							
5		1.5	9.5	185							
6		2	9	170							
7		2.5	8.5	157							
8		3	8	146							
9		3.5	7.5	137							
10		4	7	130							
11		4.5	6.5	125							
12		5	6	122							
13		5.5	5.5	121							
14		6	5	122							
15		6.5	4.5	125							
16		7	4	130							
17		7.5	3.5	137							
18		8	3	146							
19		8.5	2.5	157							
20		9	2	170							
21		9.5	1.5	185							
22		10	1	202							
23		10.5	0.5	221							
24		11	0	242							
25											
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What is the maximum total area?

What are the side lengths of square1 and square2?

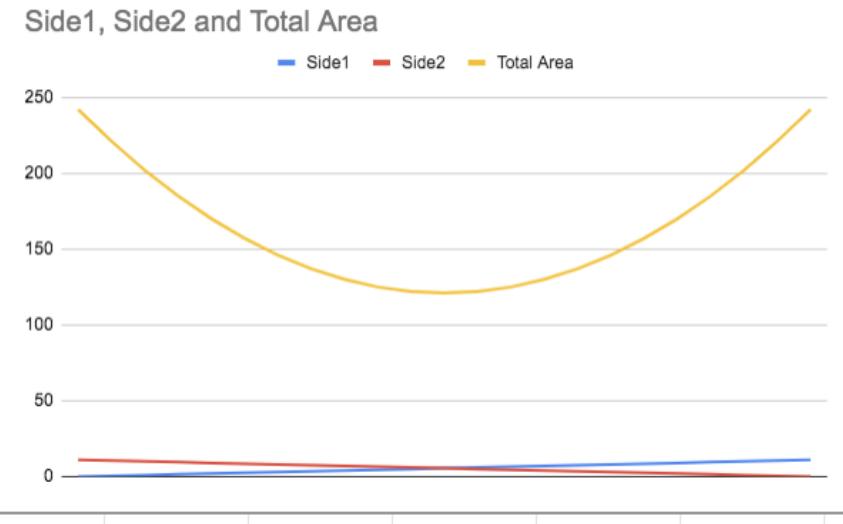
What is the minimum total area?

What are the side lengths of square1 and square2?

Are there any other possible total areas?

How do you know?

What happens if I change the length of AB to 40?





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Sample Scratch solution

The Scratch script starts with an **when green flag clicked** hat. It initializes variables: **linelengthAB** to 22, and **Side Lengths Square1**, **Side Lengths Square2**, and **Total Area** lists. The script then enters a **repeat until** loop with the condition **Side2 < 1**. Inside the loop, it adds **Side1** to **Side Lengths Square1** and **Side2** to **Side Lengths Square2**. It then adds the areas of the four squares to the **Total Area** list using the formula $2 * Side1 * Side1 + 2 * Side2 * Side2$. After each iteration, it increments **Side1** by 1 and decrements **Side2** by 1. The script concludes with a **repeat until** control block and a **stop script** control block.

Initialize variables

linelengthAB is the given length of line AB (22)
Side Lengths Square1 is a list to store possible lengths of the first set of squares
Side Lengths Square2 is a list to store possible lengths of the second set of squares
Total Area is a list to store possible total areas of all four squares
** Note: list element n in each list go together as a solution

Side1 - start with length 1 for the length of the first square
Set **Side2** using the relationship ($sidelengthAB=2*side1+2*side2$)

Find a set of possible areas - students may start at 1 for side 1 and repeat by adding 1 each time until Side2 goes below 1. Or they may increase by a smaller or greater amount. Concept of density/infinite solutions.

Add the sideLengths for A and B to their lists
Calculate and add the total area of all 4 squares to the list

Increase Side1
Decrease Side2
** Students may notice that when they increase side1, side2 decreases by the same amount. Or they may recalculate side2 using the relationship as above.

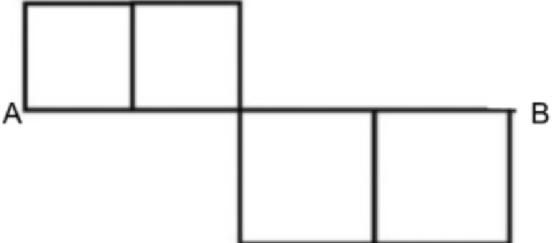
What is the maximum total area?
What are the side lengths of square1 and square2?
What is the minimum total area?
What are the side lengths of square1 and square2?
Are there more possible areas? How do you know?



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Sample Scratch solution output

linelengthAB 22

A  B

1	1
2	2
3	3
4	4
5	5
6	6

+ length 10 =

1	10
2	9
3	8
4	7
5	6
6	5

+ length 10 =

1	202
2	170
3	146
4	130
5	122
6	122

+ length 10 =



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```
when green flag clicked
  set linelengthAB to 22
  set Side1 to 0
  set Side2 to 0
  add Side1 to Side Lengths Square1
  add Side2 to Side Lengths Square2
  set AreaOf4Squares to 0
  add AreaOf4Squares to Total Area
```

Initialize variables

linelengthAB is the given length of line AB

Side1 - set the side length of the first square, what are the possible values for Side1?

Calculate the length of the second square
HINT: Use the green circle (operators) and orange circle (variables) to code a formula

```
when d key pressed
  delete all of Side Lengths Square1
  delete all of Side Lengths Square2
  delete all of Total Area
```

Add the sideLengths for A and B to their lists

Calculate the total area
HINT: use the green circle (operators) and orange circle (variables) to code the formula

You can clear your lists by pressing d

Side Lengths Square1 is a list that will store possible side lengths for the first square
Side Lengths Square2 is a list that will store possible side lengths of the second square
Total Area is a list that will store possible total Areas of the 4 squares
** Note: list element n in each list go together as a solution

```
when green flag clicked
  set linelengthAB to 22
  set Side1 to 5
  set Side2 to linelengthAB - 2 * Side1 / 2
  add Side1 to Side Lengths Square1
  add Side2 to Side Lengths Square2
  set AreaOf4Squares to 2 * Side1 * Side1 + 2 * Side2 * Side2
  add AreaOf4Squares to Total Area
```

Initialize variables

linelengthAB is the given length of line AB

Side1 - set the side length of the first square, what are the possible values for Side1?

Calculate the length of the second square
HINT: Use the green circle (operators) and orange circle (variables) to code a formula

```
when d key pressed
  delete all of Side Lengths Square1
  delete all of Side Lengths Square2
  delete all of Total Area
```

Add the sideLengths for A and B to their lists

Calculate the total area
HINT: use the green circle (operators) and orange circle (variables) to code the formula

You can clear your lists by pressing d

Side Lengths Square1 is a list that will store possible side lengths for the first square
Side Lengths Square2 is a list that will store possible side lengths of the second square
Total Area is a list that will store possible total Areas of the 4 squares
** Note: list element n in each list go together as a solution

linelengthAB 22
Side1 5
Side2 6
AreaOf4Squares 122



Tool Board